

# King-Probe Liquid Water Content (PLWCC)

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RAF Algorithm Review

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## Development and Principle of Operation

King et al. 1978

- Hot wire maintained at constant resistance (temperature)



$$P = \pi l \lambda (T_w - T) \text{Nu} + l d V \chi [L_v + c_w (T_w - T)]$$

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Extra heat required



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- Estimate heat dissipated in dry air



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## Development and Principle of Operation

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### Basic Equation:

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- Remainder is heat required for vaporization



$$P = \pi l \lambda (T_W - T) \text{Nu} + l d V \chi [L_v + c_w (T_w - T)]$$

# Current Processing

## Processing Steps

- 1 Find mean  $T$
- 2 Find the thermal conductivity, viscosity, and dry-air density:
- 3 Calculate the Reynolds number:
- 4 Find the Prandtl Number
- 5 Find the dry-air term:
- 6 Find LWC from residual
- 7 Correct for baseline offset

## Associated Equations

Calculate mean temperature between wire and air temperature, for use in determining heat-transport properties:

$$T_m = (T_W + T)/2$$

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### Associated Equations

$$\lambda = 5.8 \times 10^{-5} \frac{398}{120 + T_m} \left( \frac{T_m}{T_0} \right)^{3/2}$$

$$\mu = 1.718 \times 10^{-4} \frac{393}{120 + T_m} \left( \frac{T_m}{T_0} \right)^{3/2}$$

$$\rho_d = \frac{p}{2870.5 T_m}$$

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## Associated Equations

$$\text{Re} = \frac{\rho Vd}{\mu}$$

where  $\rho$  is the air density,  $V$  the true airspeed,  $d$  the diameter of the sensing wire, and  $\mu$  the viscosity of air



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### Associated Equations

$$\text{Pr} = c_p \mu / \lambda$$
$$\simeq 0.735$$

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### Associated Equations

$$P_{dry} = 0.26 \text{Re}^{0.6} \text{Pr}^{0.37} \left( \frac{\text{Pr}}{\text{Pr}_w} \right)^{0.25} \\ \times \pi / \lambda (T_w - T) / 0.239$$

$$\text{Nu} = 0.26 \text{Re}^{0.6} \text{Pr}^{0.37} \left( \frac{\text{Pr}}{\text{Pr}_w} \right)^{0.25}$$

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$$\chi = \frac{10^6 \times 0.239(P - P_{dry})}{l d V [597.3 + 373.16 - T]}$$

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## Critique of Current Processing

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$$\lambda = 5.8 \times 10^{-5} \frac{398}{120 + T_m} \left( \frac{T_m}{T_0} \right)^{3/2}$$

cf., Pruppacher and Klett:

$$\lambda = 0.0238 + 0.000071(T - T_0)$$

0°C: 5.87 vs 5.69; 20°C: 6.21 vs 6.03; -30°C: 5.34 vs 5.18

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$$\text{Pr} = c_p \mu / \lambda \\ \simeq 0.735$$

Almost no variability in the atmosphere; not worth including in fits and equations.



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$$P_{dry} = \pi l \lambda (T_w - T) Nu$$

$$Nu = 0.26 Re^{0.6} Pr^{0.37} \left( \frac{Pr}{Pr_w} \right)^{0.25}$$

Doesn't fit GV measurements  
at all.

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$$\chi = \frac{10^6 \times 0.239(P - P_{dry})}{l d V [597.3 + 373.16 - T]}$$

- Matches King equation only for  $T_w = 100.01^\circ\text{C}$  (and  $c_w = 1\text{cal/g}$ )
- Neglects T dep. of  $L_v(T)$ .

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- OK, but would be better to correct Nu relationship and hence  $P_{dry}$  directly.

## Equation for the Nusselt Number

### Representation in terms of Re

$$\text{Re} = \frac{\rho_d V d}{\mu(T_m)} = \frac{\rho V d}{R_d T_m \mu(T_m)}$$

$$\text{Nu} = A_N \text{Re}^\alpha \text{ or } \log(\text{Nu}) = \log(A_N) + \alpha \log(\text{Re})$$

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## Some Research-Project Problems

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- ③ Flow distortion around the wings and probes may cause size-dependent sorting of droplets with associated size-dependent increases or decreases in concentration.

## Some Research-Project Problems

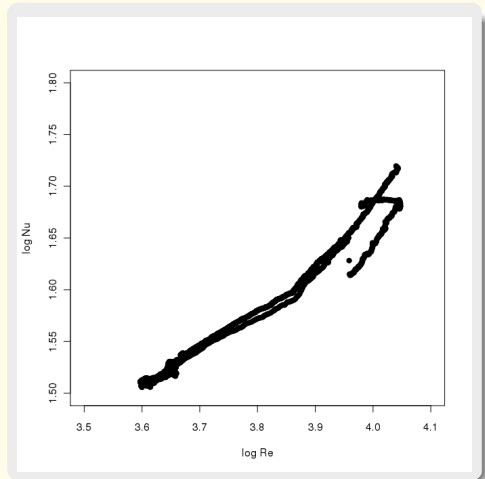
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  - Flow effects may vary with angle-of-attack or sideslip angle.
- 2 For the GV especially, dynamic heating increases the temperature of the air in thermal contact with the sensor, affecting the heat transfer from the wire.
- 3 Flow distortion around the wings and probes may cause size-dependent sorting of droplets with associated size-dependent increases or decreases in concentration.
- 4 Flow at an angle to the sensor will reduce the heat loss (approximately as the cosine of the flow angle relative to the axis of the sensing wire).

# GV Data (PREDICT)

## Examples

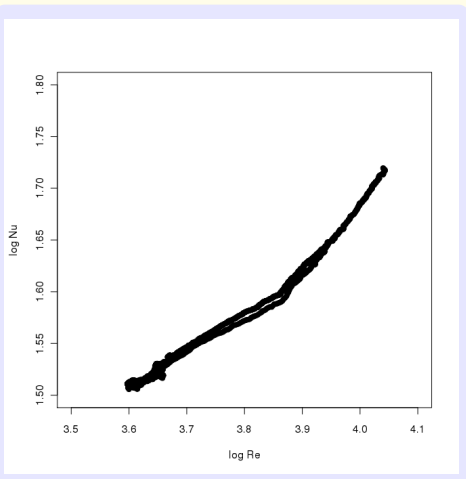
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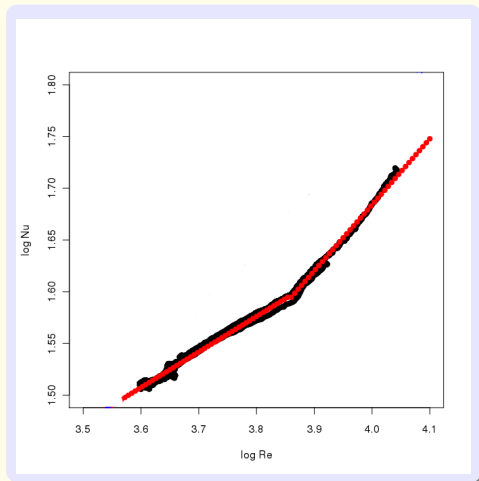




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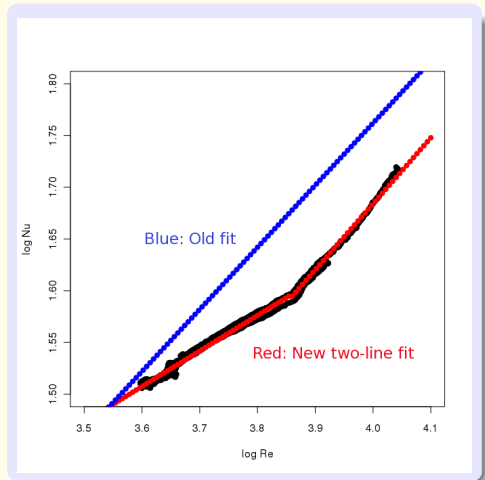
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# GV Data (PREDICT)

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- 4 Comparison to current parameterization



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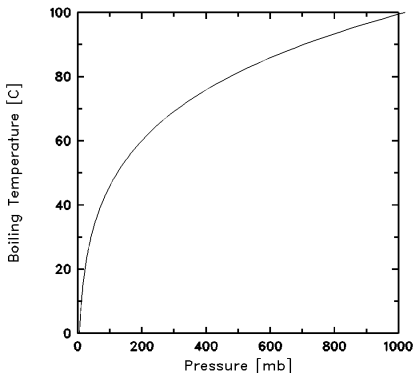
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 $T_{shed} < T_w < T_{BP}$
- Wire temperature now 130–165°C
- $T_{BP}$ : Reason for 100°C in equations
- However, the boiling point decreases with pressure:



# Baseline Removal

## Current Technique

- 1 When out-of-cloud:
  - calculate running-average LWC (30 s)
  - each sample: require 2-s average to be < threshold
  - running average: not necessarily a contiguous sample

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## Advantage (small)

If T and TAS vary in cloud:

- Updated Nu represents these variations via Re
- Updated zero does not

# GV Results

## PREDICT

- 1 PREDICT: hard to find real LWC regions
- 2 still can do calculations where  $CONCD > 0.1$
- 3 Result: Some compensating changes:
  - most important:  $L_v(T)$  higher,  $c_w T_{BP}$  lower
  - net change in example: about 4% increase,  $< 0.002$  RMS

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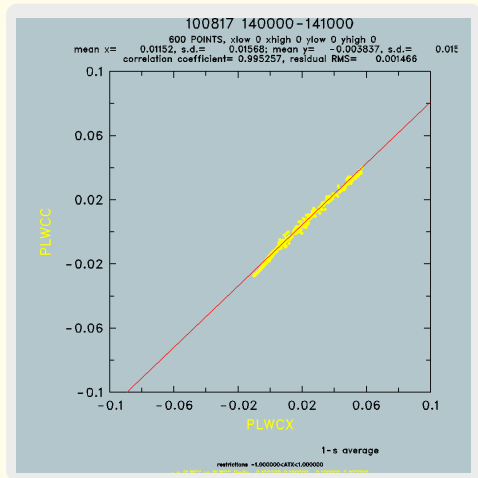
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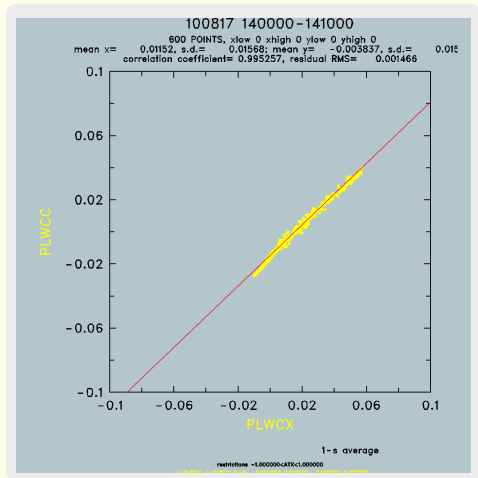
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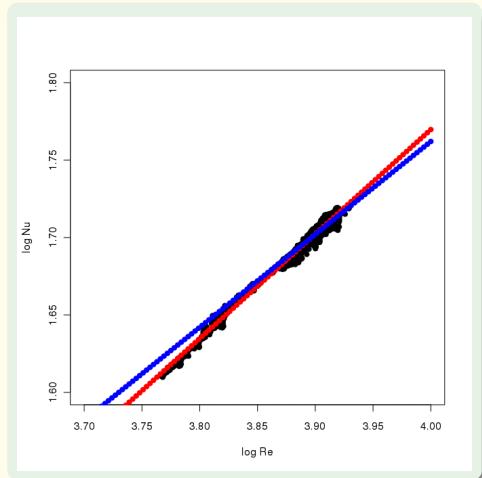
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## VOCALS

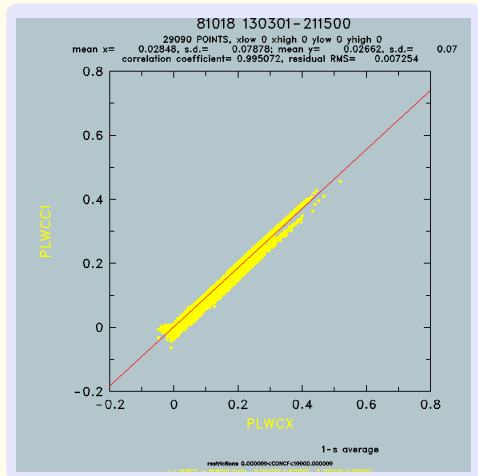
- 1 Determine Nu-Re relationship according to new scheme
  - new fit (red) is close to blue curve (blue)
- 2 Result: changes are larger:
  - increase of about 9%, mostly from  $L_v$  change
  - some scatter arises from zero correction:  $<0.01 \text{ g m}^{-3}$ .



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- 7 Change to MKS units for consistency.
- 8 Change the baseline correction to update the dry-air term.