Using a Complementary Filter

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RAF Algorithm Review

09/28/2011

The Objective

Combining Independent Measurements, e.g., of Aircraft Velocity

Resulting Variables: {VEWC, VNSC}

Inertial reference system (accelerometers and angles)

- good short-term precision
- good short-period response
- drift over longer periods
- Schuler oscillation that arises from error in initial alignment

Variables: {VEW, VNS}

Global Positioning System (absolute position differentiated)

- good absolute accuracy (without drift or oscillation)
- less precise for short periods

Variables: {GVEW, GVNS}

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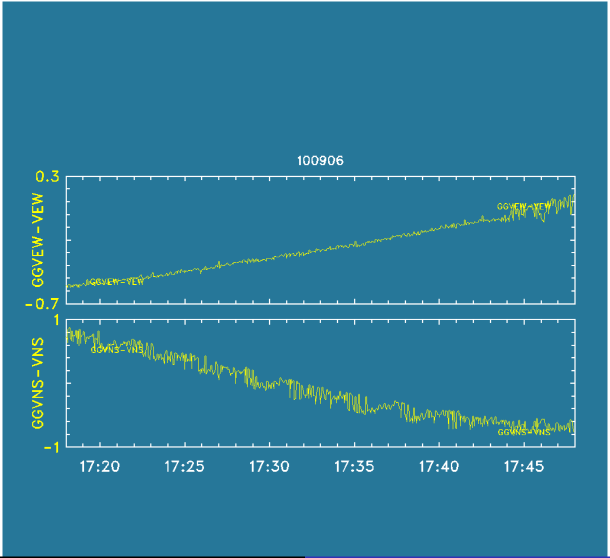
Why Correct?


100906
0.3
-0.7
GGVEW-VEW
GGVEW-VEW
GGVEW-VEW
1
-1
GGVNS-VNS
GGVNS-VNS
GGVNS-VNS
17:20
17:25
17:30
17:35
17:40
17:45

The Approach

Use the best of each:

1. High-pass filter the IRS measurements to preserve short-period response
2. Low-pass filter the GPS measurements to preserve absolute accuracy
3. Add the results

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$$\boxed{\begin{array}{l} V_{EW} = F_H(\{\mathrm{VEW}\}) + F_L(\{\mathrm{GVEW}\}) \\ V_{NS} = F_H(\{\mathrm{VNS}\}) + F_L(\{\mathrm{GVNS}\}) \end{array}}$$

The Filters

Desirable Characteristics

1. "Complementary": $F_H(v) + F_L(v) = v$ for all frequencies
2. No phase shift (suggests Butterworth filter)
3. recursive, not centered (doesn't require two passes for data processing)

The Choice Made (<1990)

- F_L=Three-pole Butterworth low-pass filter, digital
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- F_H=(1-F_L)

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Processing Code

CONSTANTS (dependent on time constant τ) :[a]

$$a = \frac{2\pi}{\tau}, \qquad a_2 = a\,e^{-a/2}\left(\cos(a\sqrt{\tfrac{3}{2}}) + \sqrt{\tfrac{1}{3}}\sin(a\sqrt{\tfrac{3}{2}})\right),$$

$$a_3 = 2e^{-a/2}\cos(a\sqrt{\tfrac{3}{2}}, \; a_4 = e^{-a}$$

```
// input x = unfiltered signal
// output returned is low-pass-filtered input
// tau determines the cutoff
// zf[] saves values  for recursion
zf[2] = -a*x + a2*zf[5] + a3*zf[3] - a4*zf[4];
zf[1] = a*x + a4*zf[1];
zf[4] = zf[3];
zf[3] = zf[2];
zf[5] = x;
return(zf[1] + zf[2]);
```

Application to Wind Measurements

Treatment when GPS is bad

- Need to extrapolate smoothly to avoid discontinuities in measurements
- The approach:
    1. Fit good data to find $\mathbf{\Delta} = (\mathbf{u}_G - \mathbf{u}_I)$ (e.g., GVEW-VEW): $\Delta = c_1 + c_2 \sin(\Omega t) + c_3 \cos(\Omega t)$ – i.e., Schuler oscillation. Emphasize latest measurements: Let matrix for fit decay exponentially.
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    2. If GPS data become unavailable, take steps to avoid discontinuities:
        1. If a valid fit has been accumulated, it is used to continue to correct $\mathbf{u_i}$ using Δ as above (i.e., $\mathbf{u}' = \mathbf{u_i} + \acute{}$)
        2. If no valid fit has been accumulated, let latest correction decay exponentially (336 s).

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More Details (Wind)

Transitions

- When GPS reception is lost, with fit accumulated that predicts Δ (the correction to {VEW,VNS} to match {GVEW,GVNS}), and with $\eta = 0.997$,{VEWC}={VEW}+Δ' where:

$$\Delta' = \eta(\{VEWC\}_0 - \{GVEW\}_0) + (1-\eta)\Delta$$

$$\{VEWC\} = \{VEW\} + \Delta'$$

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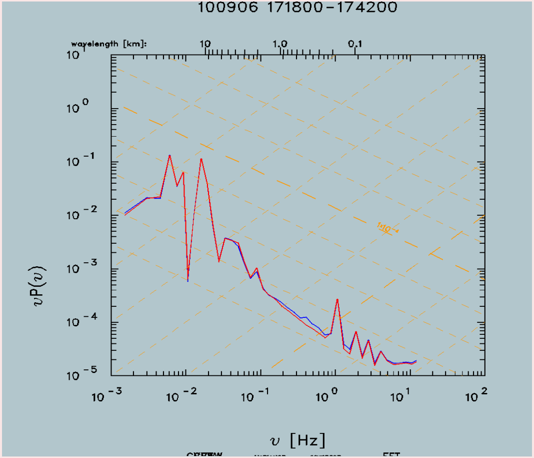
Choice of Time Constant?

Guidelines:

- Should be where both the GPS and IRS give the same signal, so that details of the filter aren't important
- Once way to make this choice: Examine variance spectra for GPS and IRS variables, and coherence:
    - Want variance spectra to be the same in the region of transition, confirming similar magnitude signals there
    - Want coherence = 1 and phase = 0

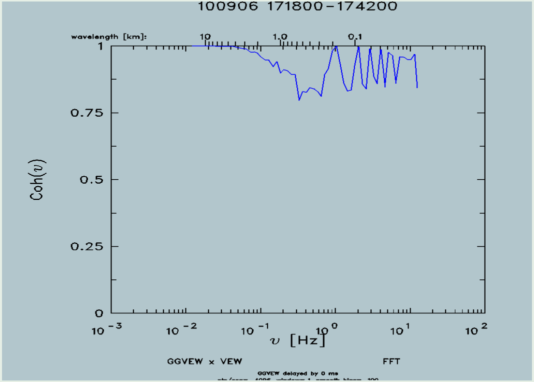
Variance Spectra for GGVEW (blue) and VEW (red)

PREDICT Flight 12, flight segment at 10,000 ft



Coherence GGVEW/VEW

Coherence



Corresponding Phase: Solidly zero throughout the frequency range. Might shorten to 100 s or less? >20 s appears acceptable.

Application to Position

The Corrected Position (LATC,LONC):

1. Initialize $\mathbf{x}' = \mathbf{x}$ at the start of a flight or after any large change.
2. Integrate forward using $\mathbf{u}'$, which gives good short-term accuracy
3. Adjust to the GPS position exponentially, with about 100-s time constant, using $\mathbf{x}' \mathrel{+}= \eta(\mathbf{x}_G - \mathbf{x}_I)$ with $\eta = 2\pi/600s$
4. For periods when the GPS signal is lost, use a procedure analogous to that for velocity:
    1. Accumulate a fit to the difference $(\mathbf{x}_G - \mathbf{x}_I)$
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Suggested Minor Changes

Suppress Decay of Corrections

- GPS is much more reliable than when this was first implemented.
- If GPS reception is lost and there is no valid fit:
  - now, the correction decays and the corrected variables move toward the IRS values
  - it would be better to retain the last offset in both signals until GPS reception returns

Correct An Abrupt Transition

- Now, when GPS measurements resume, the corrected measurements for velocity return to the complementary-filtered results abruptly.
- Better: make this transition exponentially as is now done when GPS reception is lost.

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For Both Cases (and Position):

Calculate target (GPS or fit), and update to that target.

Details

See Memo for Details and Implementation Notes