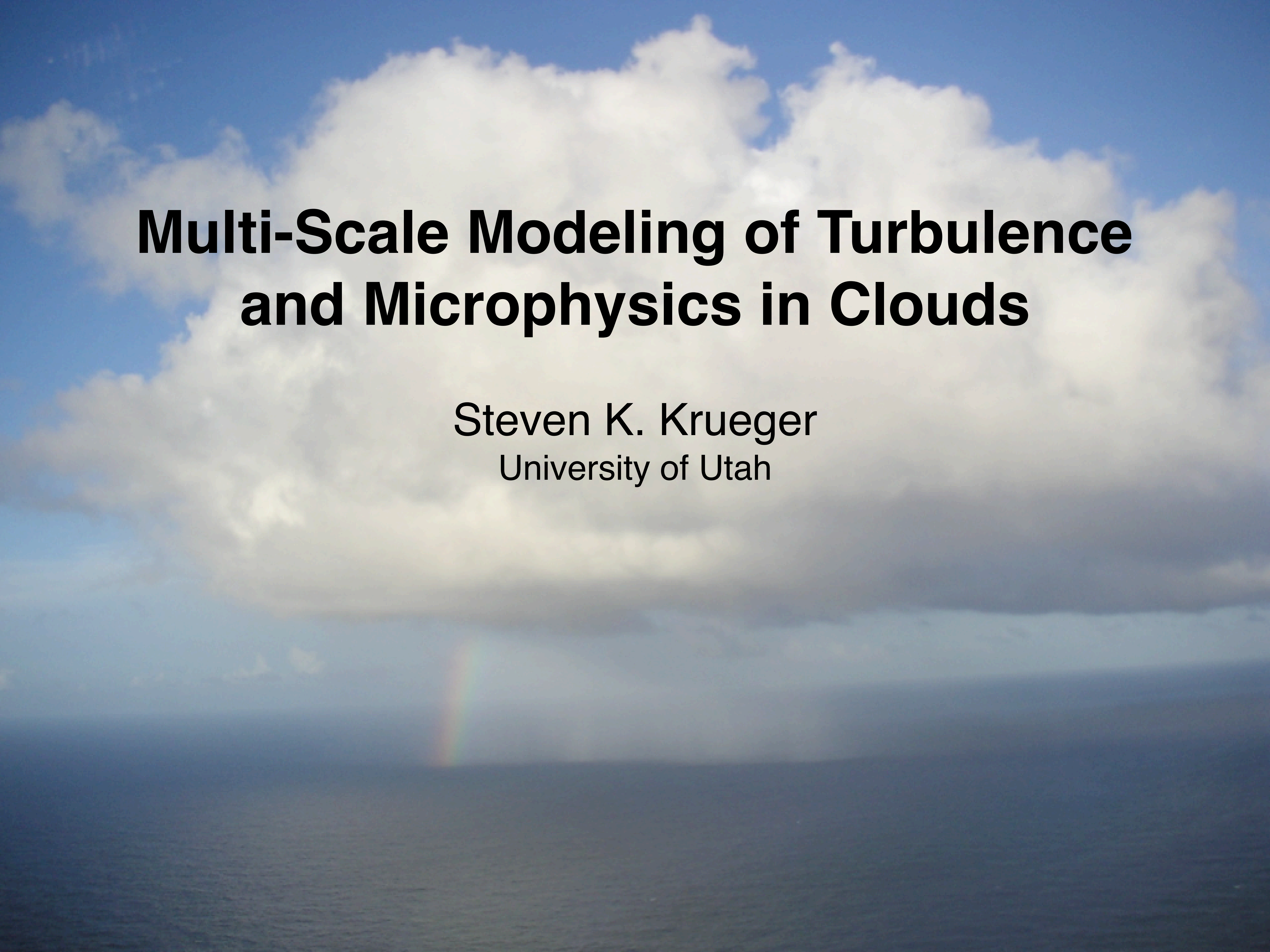
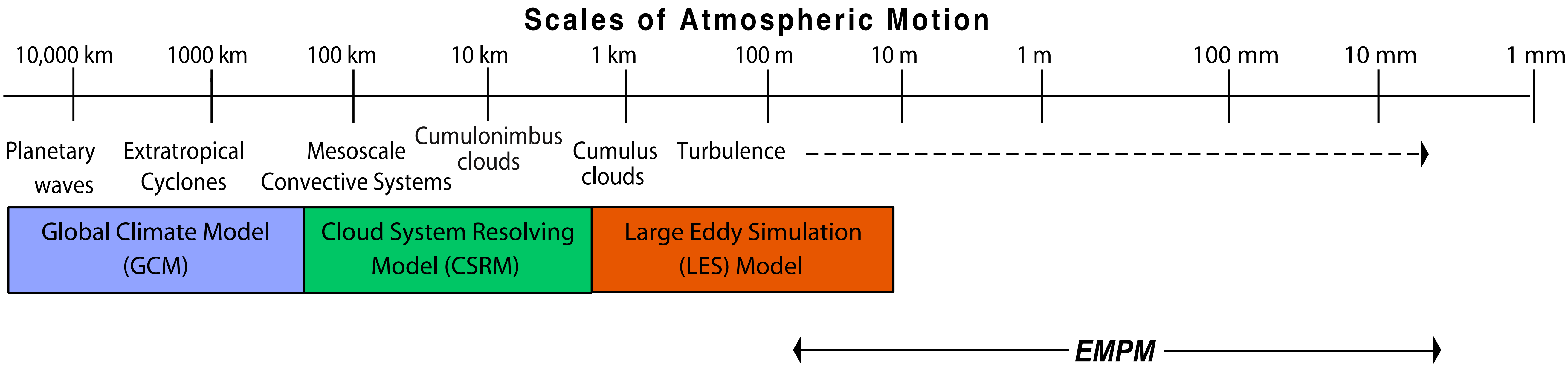


Multi-Scale Modeling of Turbulence and Microphysics in Clouds

Steven K. Krueger
University of Utah

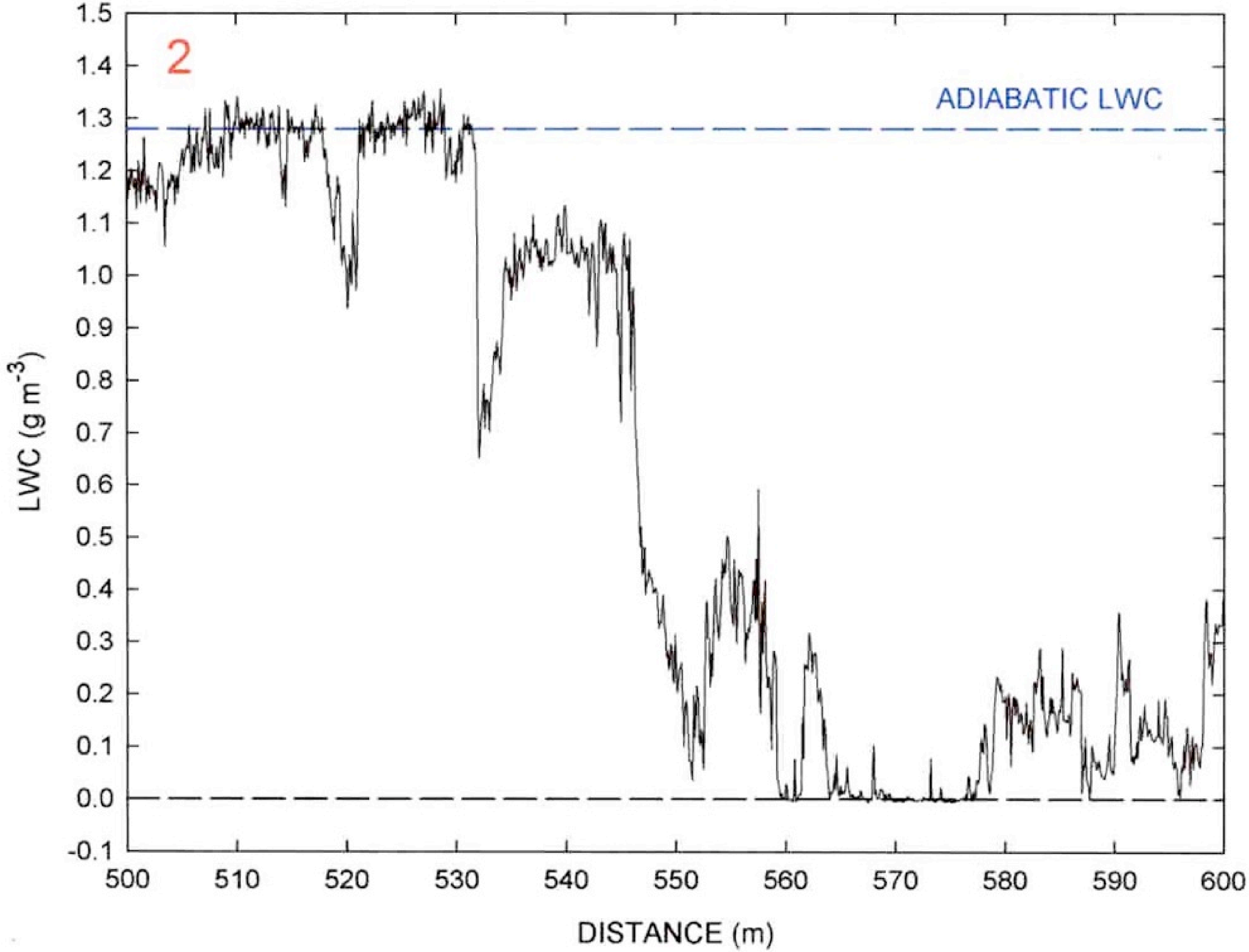


The smallest scale of turbulence is the Kolmogorov scale:

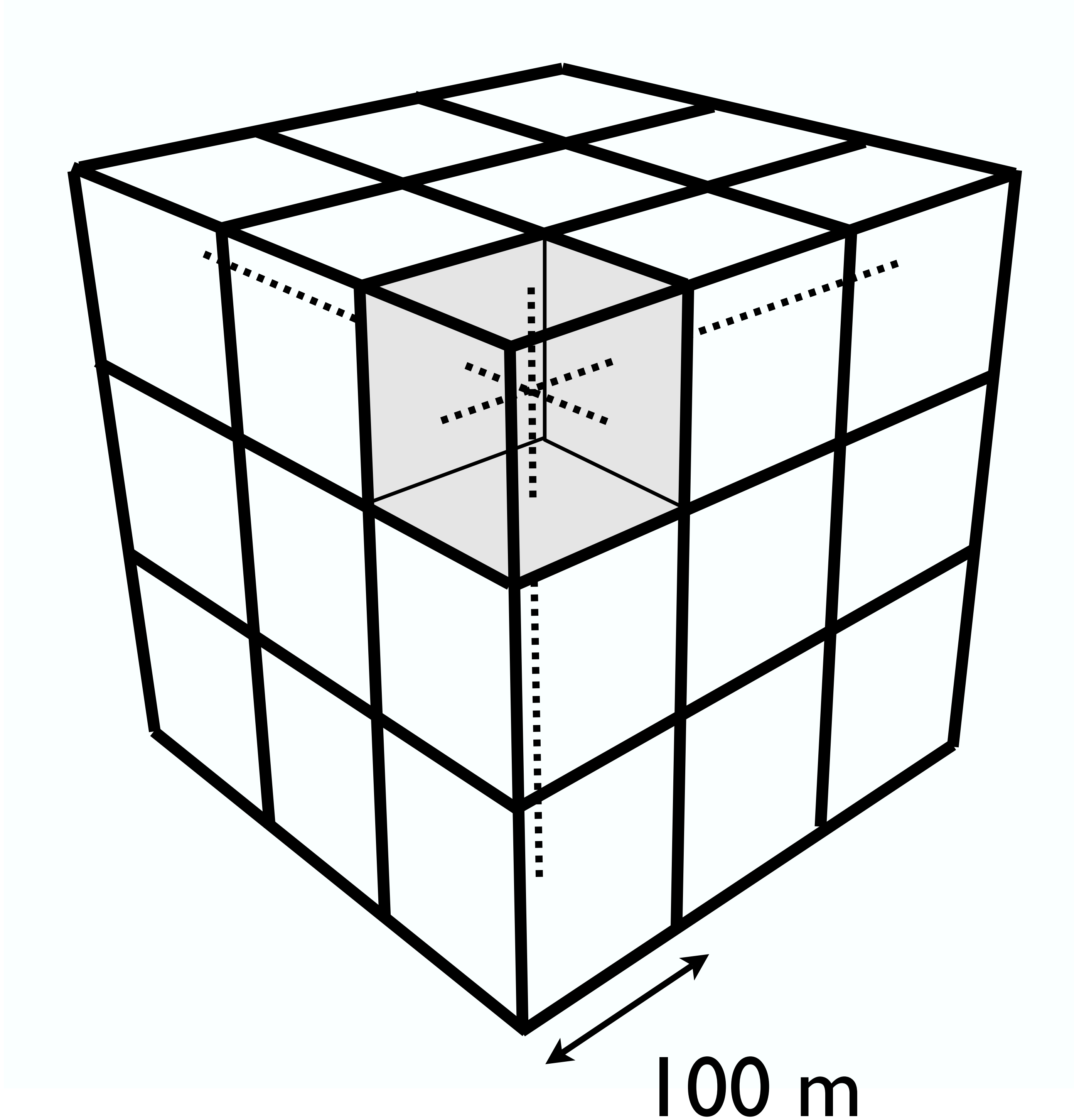
$$\eta \equiv (\nu^3/\epsilon)^{1/4}$$

For $\epsilon = 10^{-2}$ m^2 s^{-3} and $\nu = 1.5 \times 10^{-5}$ m^2 s^{-1}, $\eta = 0.7$ mm.

Aircraft Measurements of Liquid Water Content



Large-Eddy Simulation (LES) model



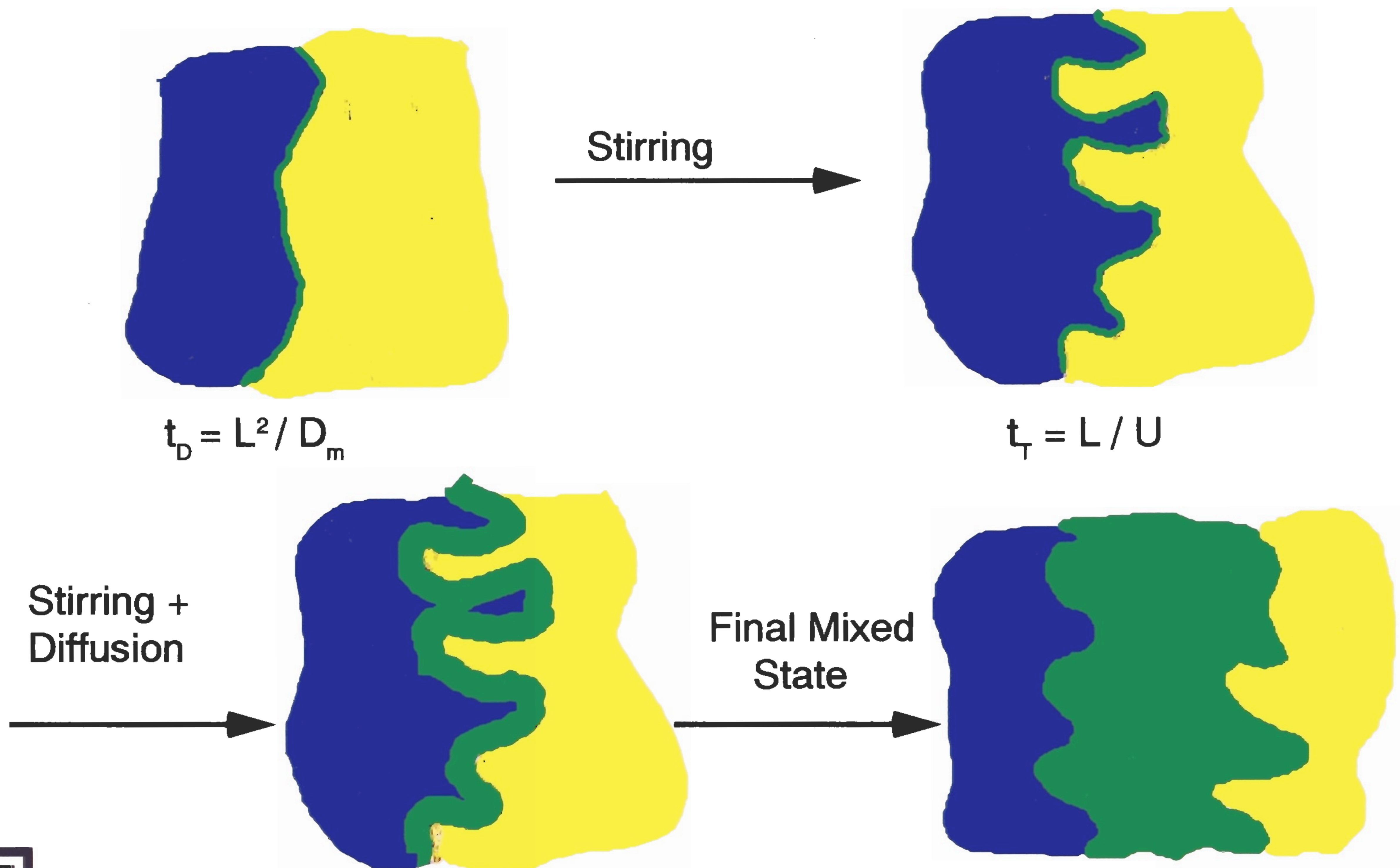
LES Limitations

- The premise of LES is that only the large eddies need to be resolved.
- Why resolve any finer scales? Why resolve the finest scales?
- LES is appropriate if the important small-scale processes can be parameterized.
- Many cloud processes are subgrid-scale, yet can't (yet) be adequately parameterized.

Subgrid-scale Cloud Processes

- SGS finite-rate mixing of clear and cloudy air slows evaporative cooling and affects buoyancy and cloud dynamics.
- SGS variability of water vapor due to entrainment and mixing broadens droplet size distribution (DSD) and increases droplet collision rates.
- SGS turbulence increases droplet collision rates.

Turbulent Mixing: Process by which a fluid with two initially segregated scalar properties mix at the molecular level



LES of passive scalar in a convective boundary layer (grid size = 20 m)

Mixing Time Scale

$$\tau = \left(\frac{d^2}{\epsilon}\right)^{1/3},$$

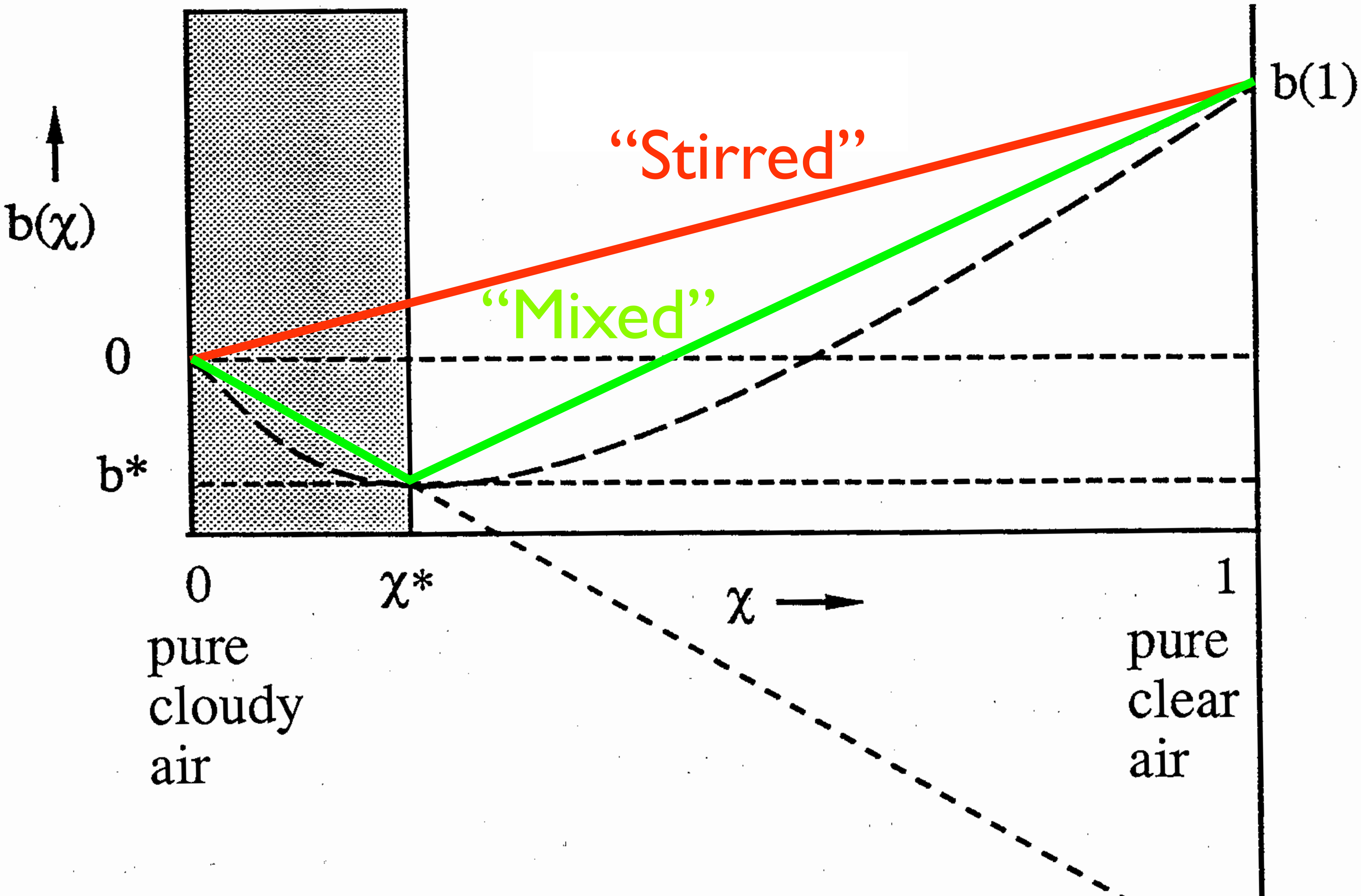
d is entrained blob size, ϵ is dissipation rate of turbulence kinetic energy.

For a **cumulus cloud,** $U \sim 2$ m/s, $L \sim 1000$ m, so $\epsilon \sim U^3/L = 10^{-2}$ m^2/s^3. For $d = 100$ m, $\tau \sim 100$ s.

Classic (instant mixing) parcel model is recovered when

- Entrained blob size, $d \to 0$
- Turbulence intensity, $\epsilon \to \infty$

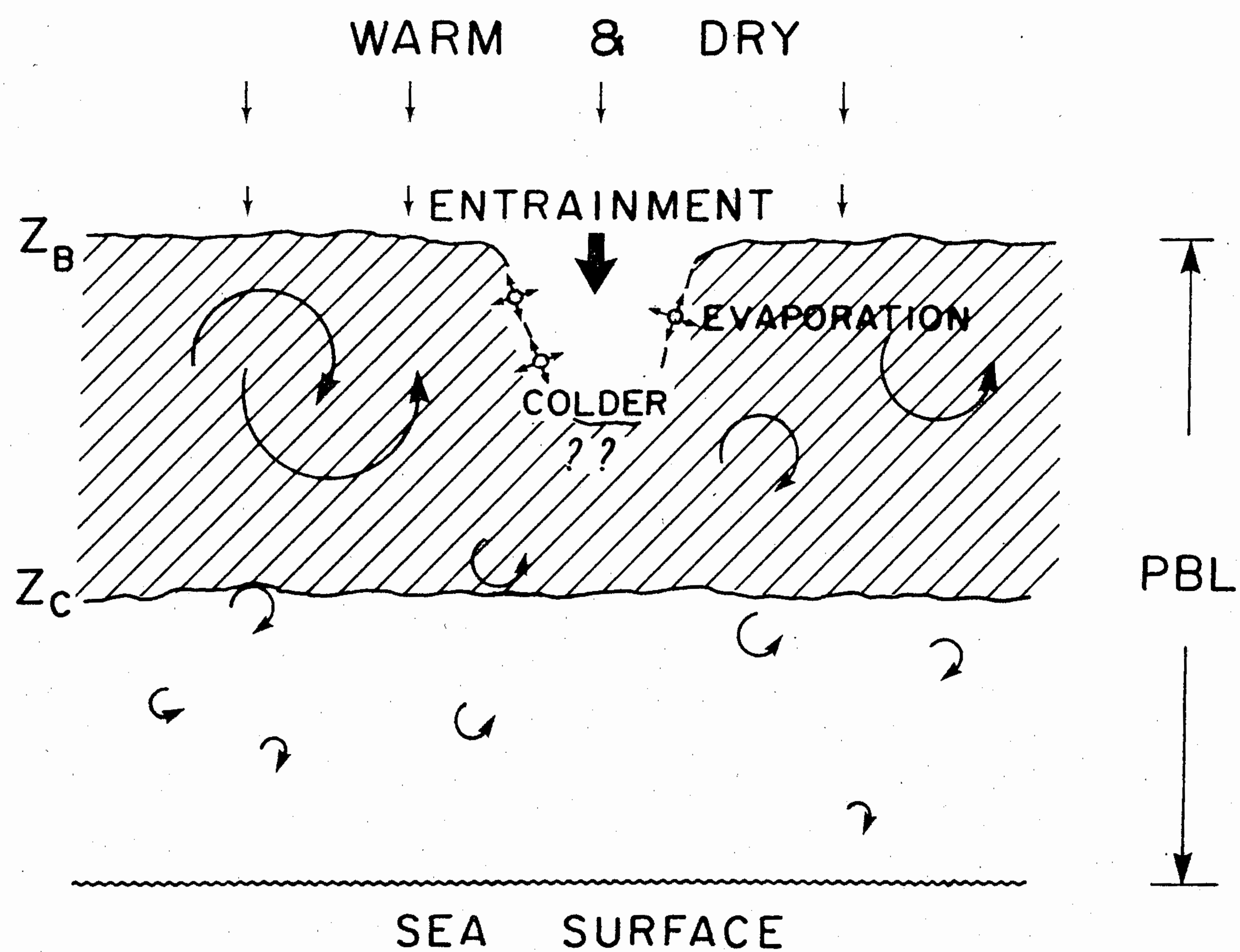
Buoyancy vs Mixture Fraction



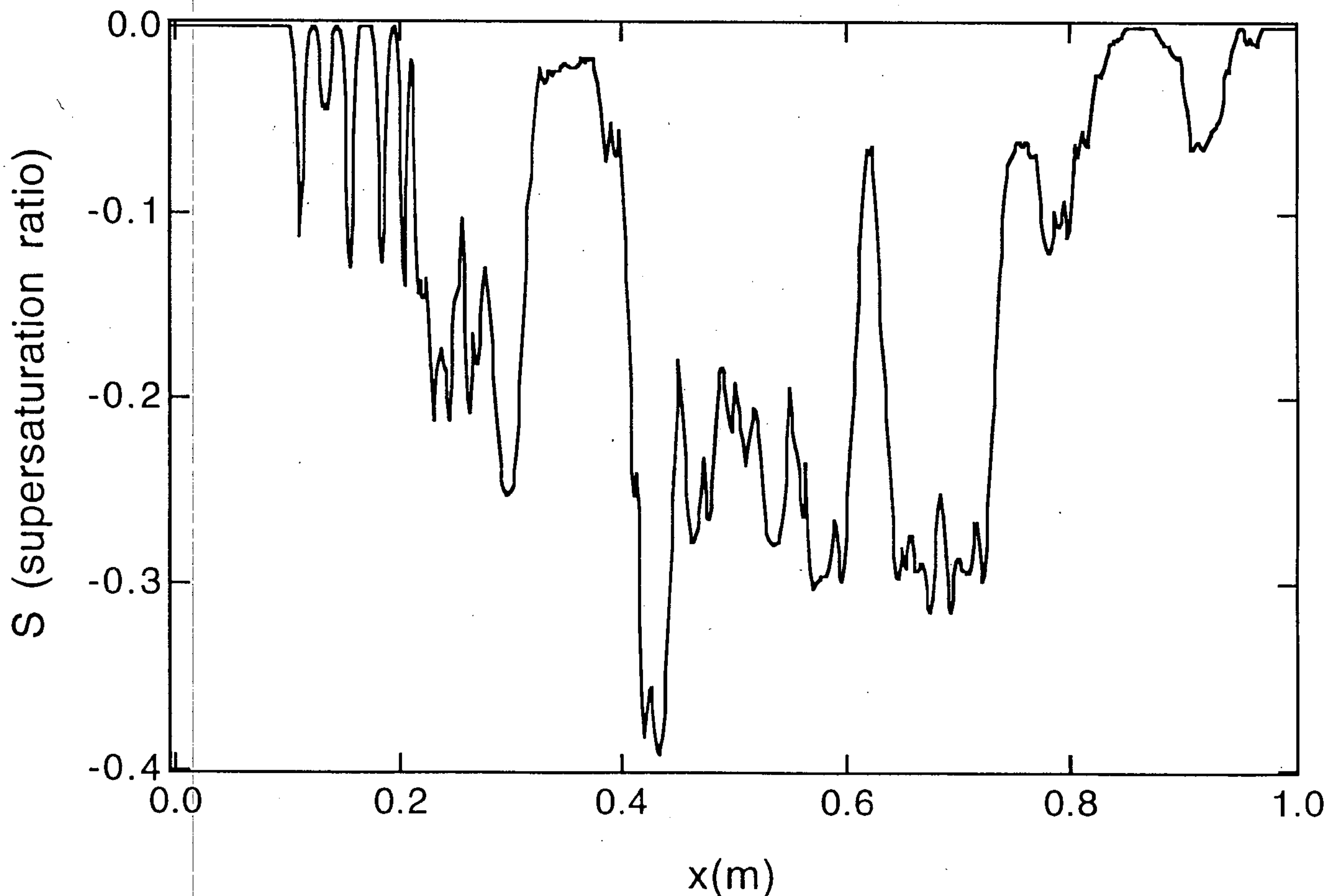
χ: mixing fraction of clear air

b(χ): buoyancy of air with mixing fraction χ

Cloud-top Entrainment Instability (CEI)



Snapshot of supersaturation ratio during mixing



An unsaturated blob is entrained

individual droplet radii

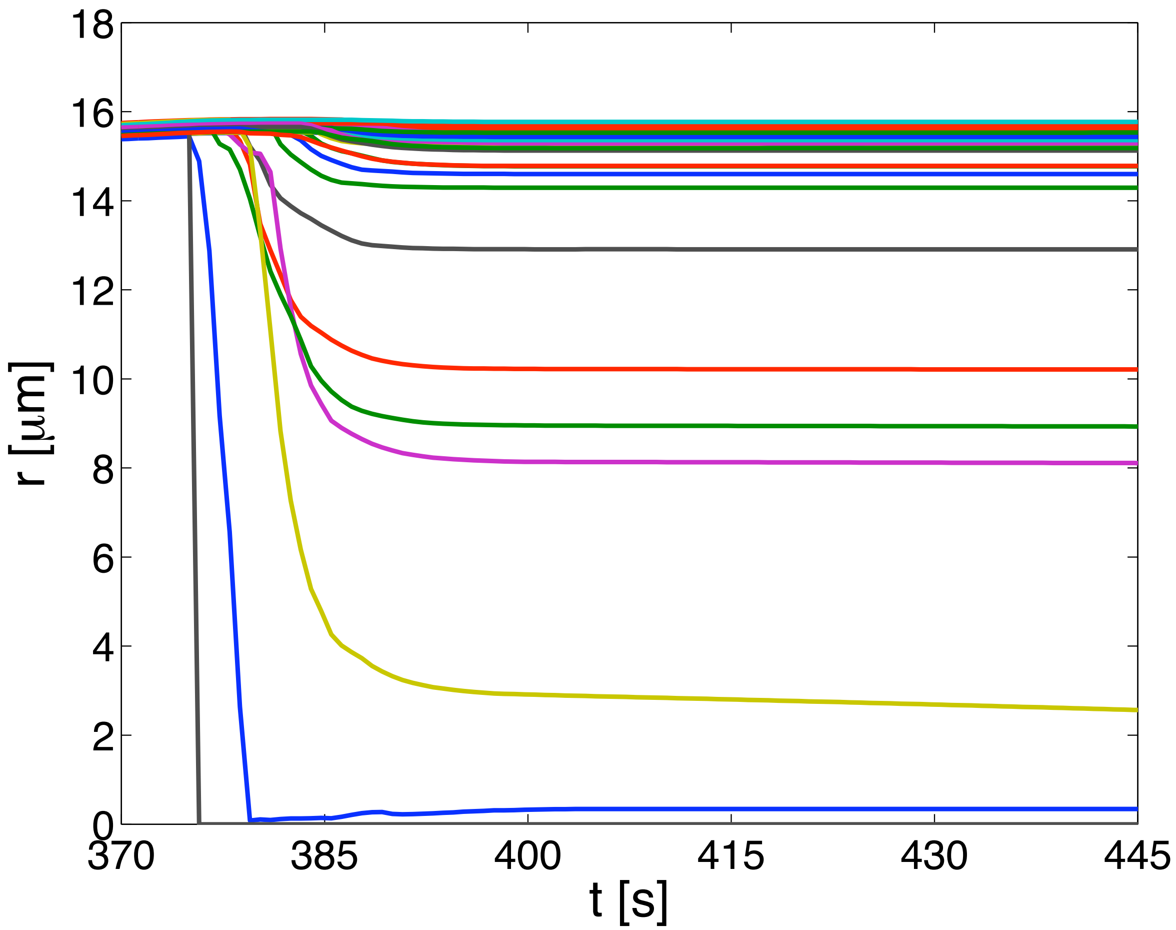


Figure 4.10: Radius histories of 30 droplets for $f = 0.1$ and $RH_e = 0.219$.

width of droplet size distribution

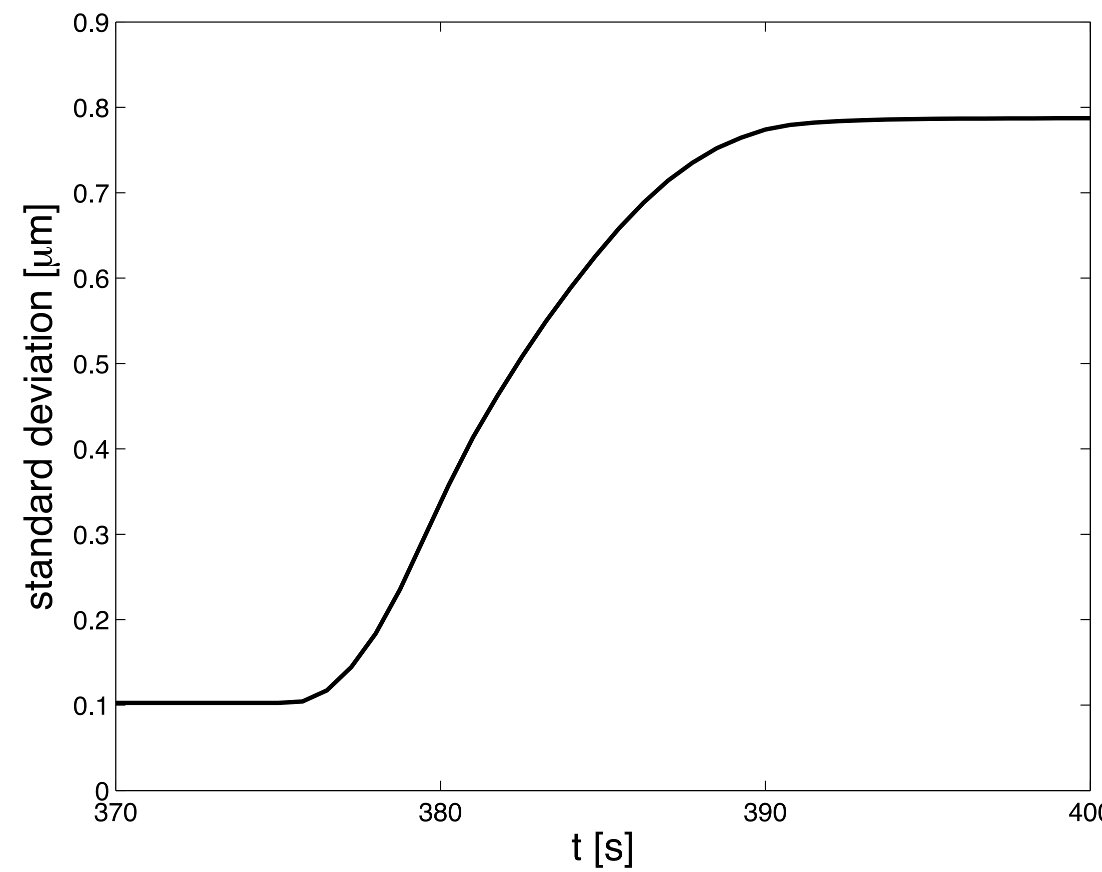
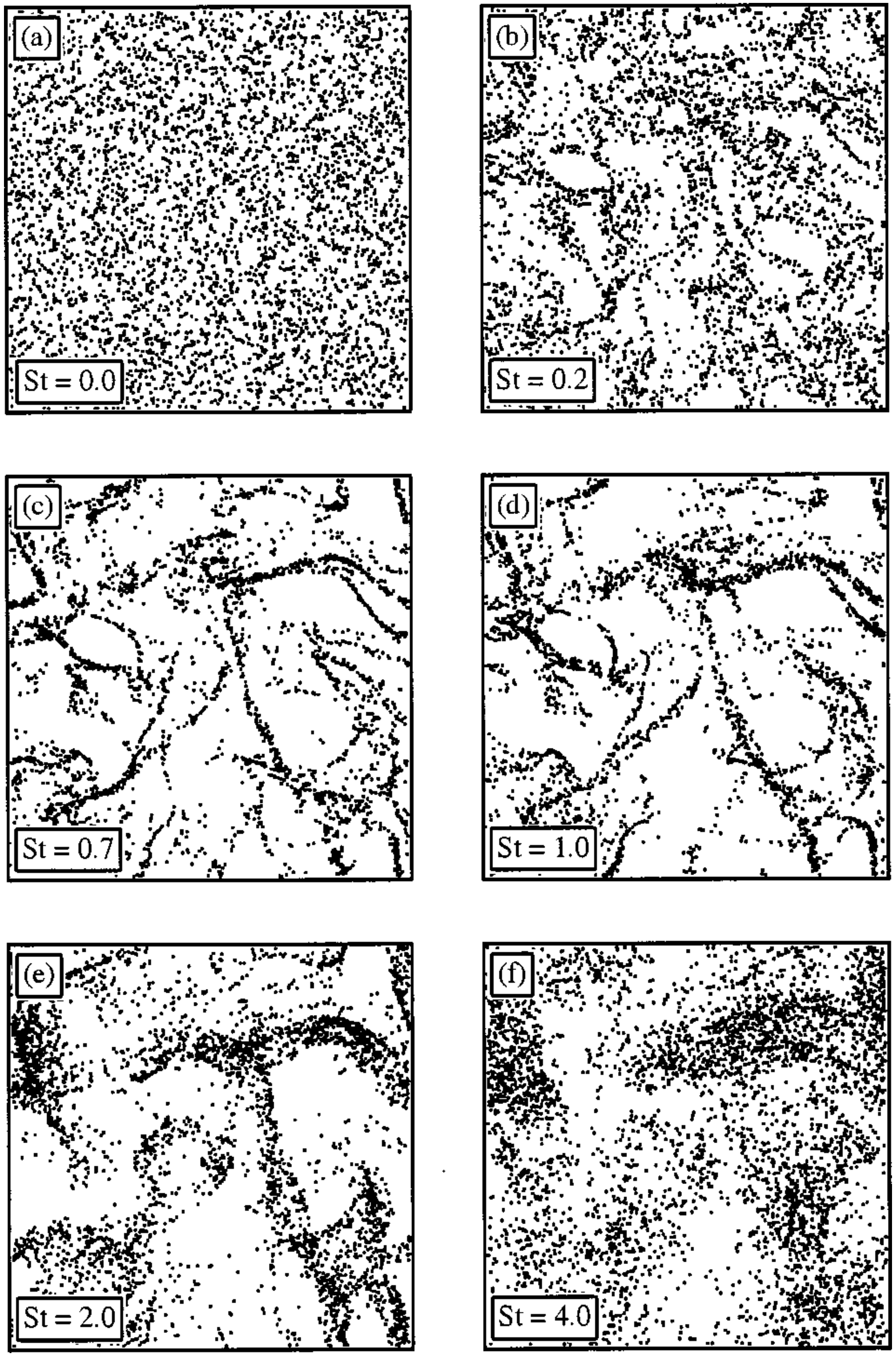


Figure 4.6: Standard deviation of the droplet radii just before entrainment until homogenization for entrainment fraction $f = 0.2$ for the control case.

Large droplets are needed to initiate collision-coalescence growth

- *Processes that may contribute to large droplet production*
  - Entrainment and mixing of unsaturated air
  - Droplet clustering due to turbulence
  - Giant aerosols

Clustering of inertial particles in turbulence increases collision rates



Direct numerical simulation results from Reade & Collins (2000)

Parameterization of SGS Cloud Processes in LES

- SGS mixing is instantaneous in most LES.
- SGS variability does not affect DSD in any LES.
- SGS turbulence affects droplet collision rates in very few LES.

Parameterization of SGS Cloud Processes in LES
(and how to improve, v. 1)

- SGS mixing is instantaneous in most LES. (Decrease grid size or estimate SGS PDF.)
- SGS variability does not affect DSD in any LES. (Decrease grid size.)
- SGS turbulence affects droplet collision rates in very few LES. (Modify collision kernel.)

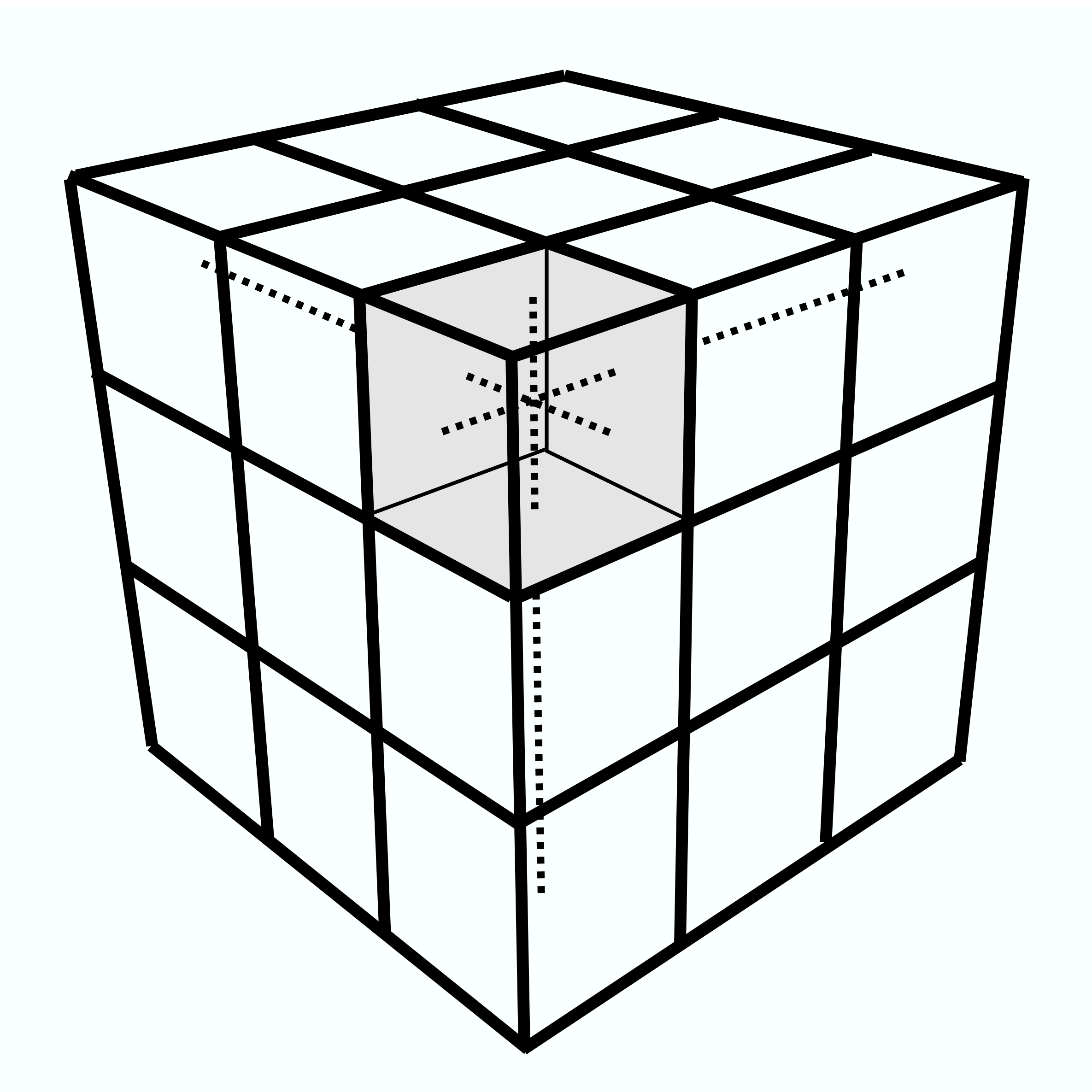
How to resolve the small-scale variability?

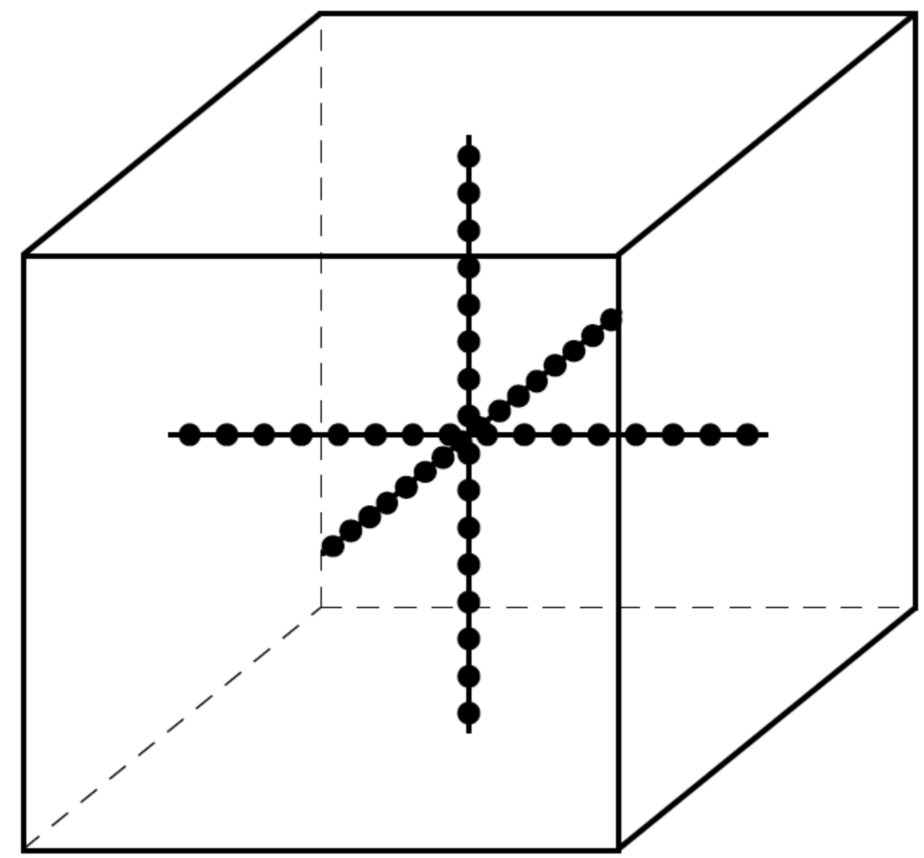
- Decrease LES grid size?
  - To decrease LES grid size from 100 m to 1 cm would require 10^{12} grid points per $(100\ m)^3$ and an increase in CPU time of 10^{16}.
  - *This is not possible now or in the forseeable future.*

How to resolve the small-scale variability?

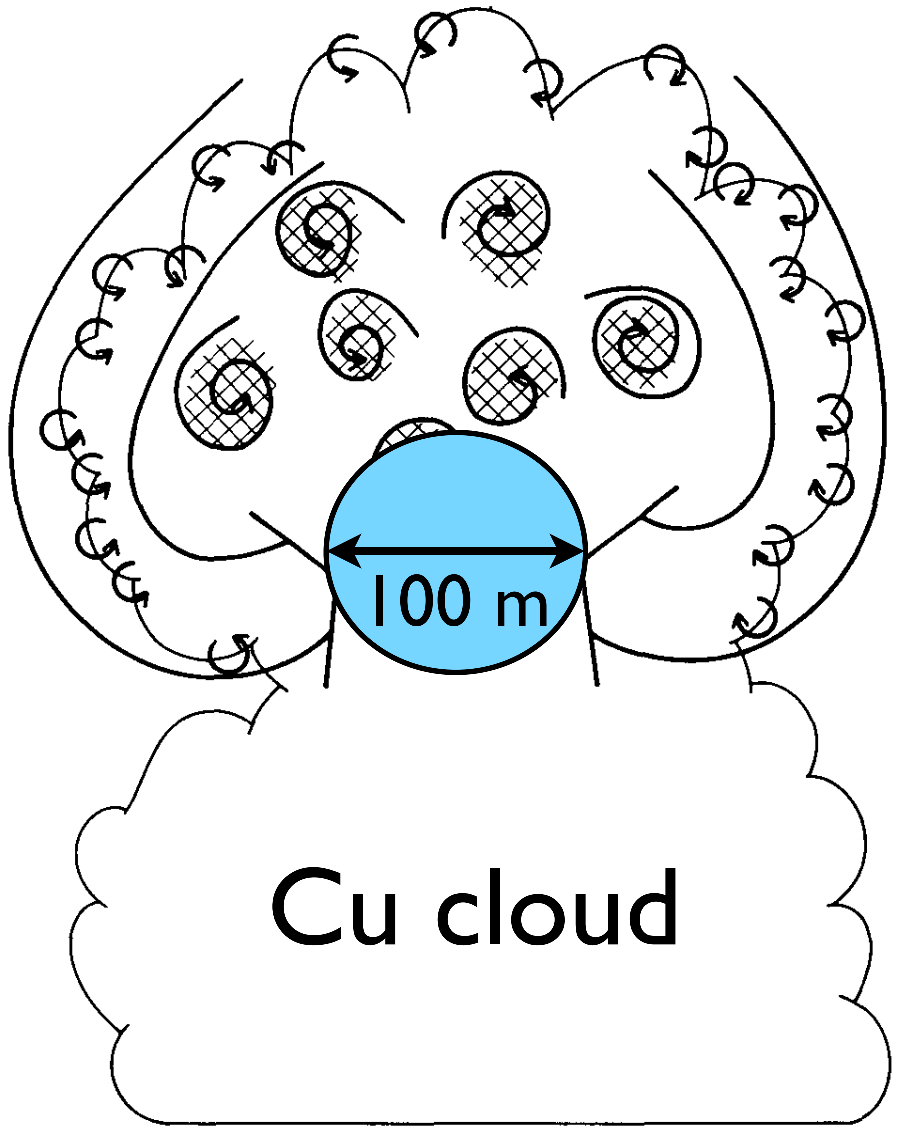
- Decrease dimensionality from 3D to 1D?
  - To decrease grid size from 100 m to 1 cm would require only 10^4 grid points points per $(100\ m)^3$.
  - *This is feasible now.*

LES with 1D subgrid-scale model

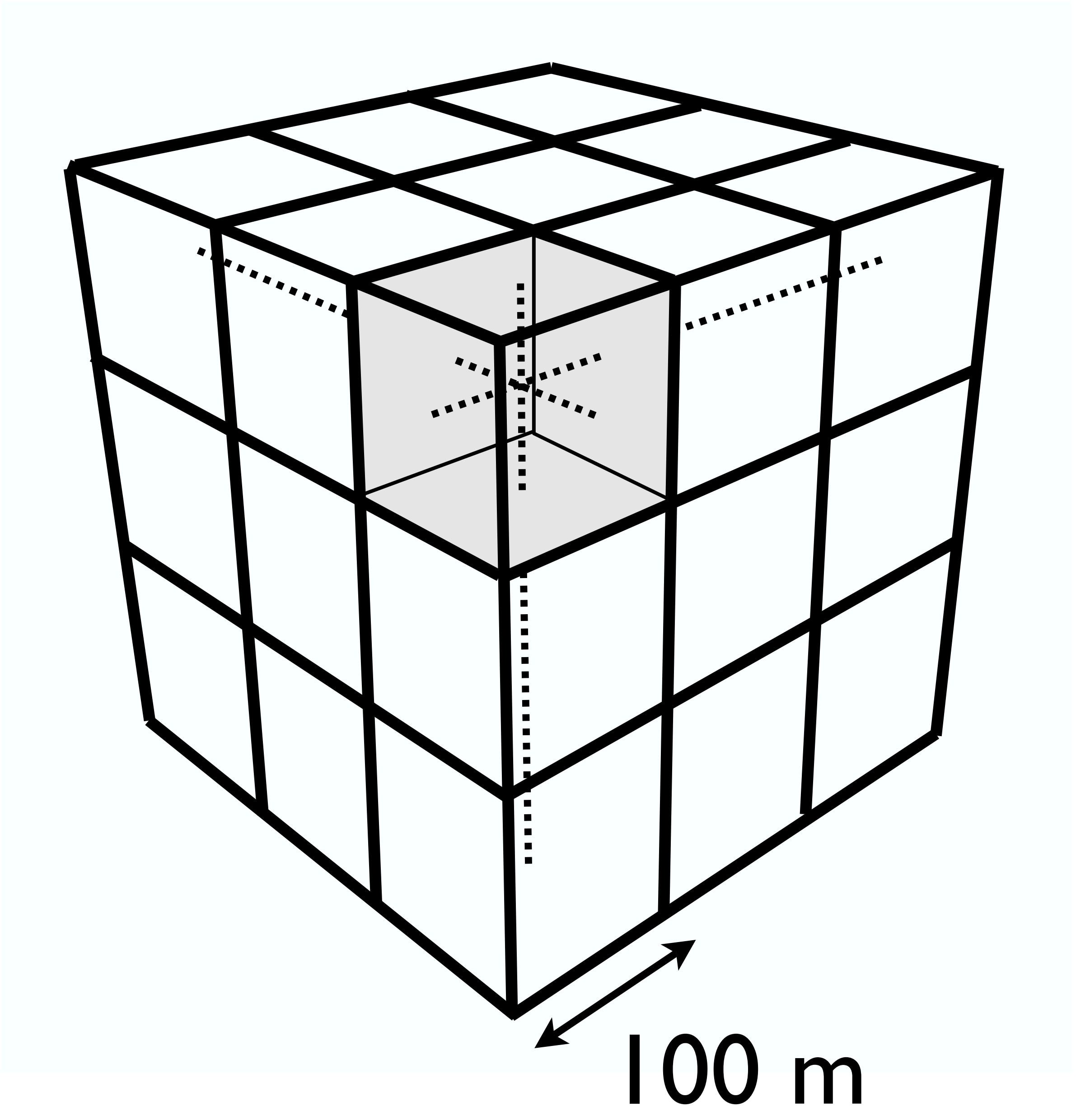




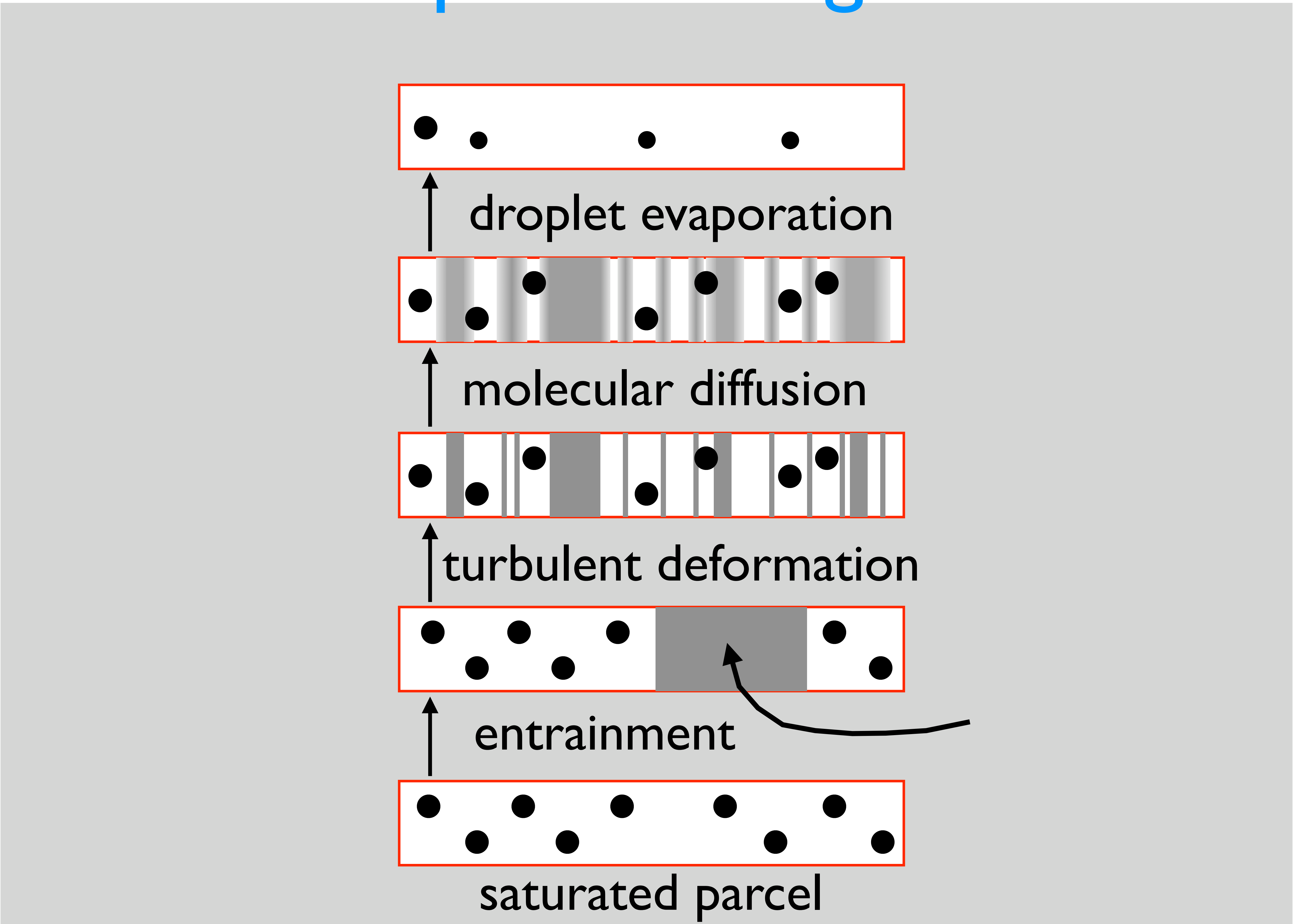
Parcel model



Large-Eddy Simulation (LES) model



1-D explicit mixing model



Turbulent Motion is Represented by Applying Maps

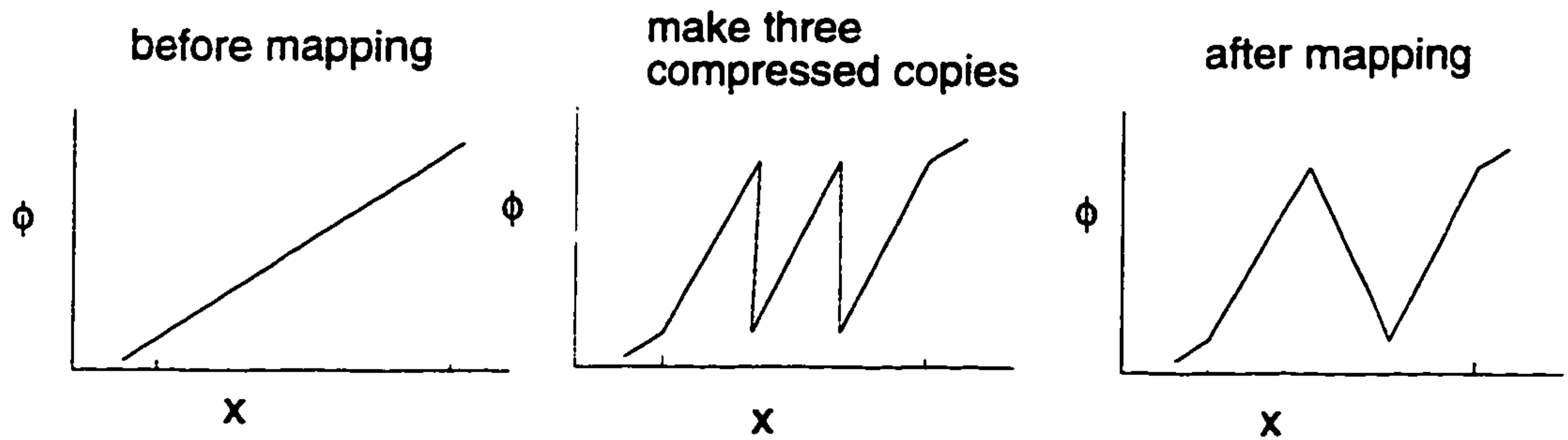


Figure 2.1. Schematic diagram of a triplet mapping event.

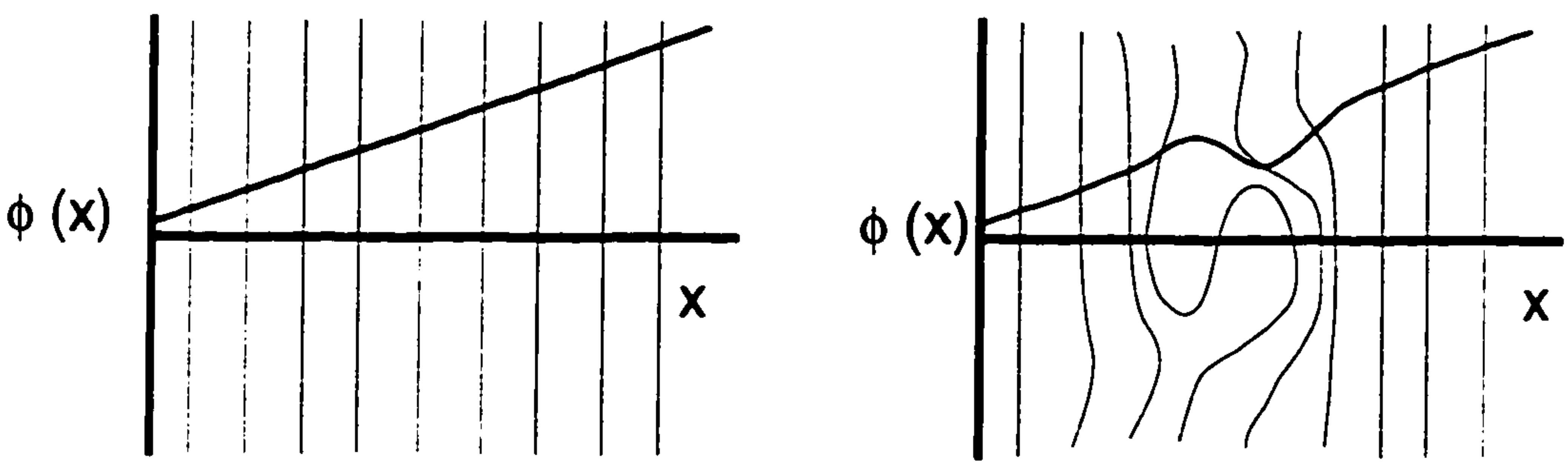


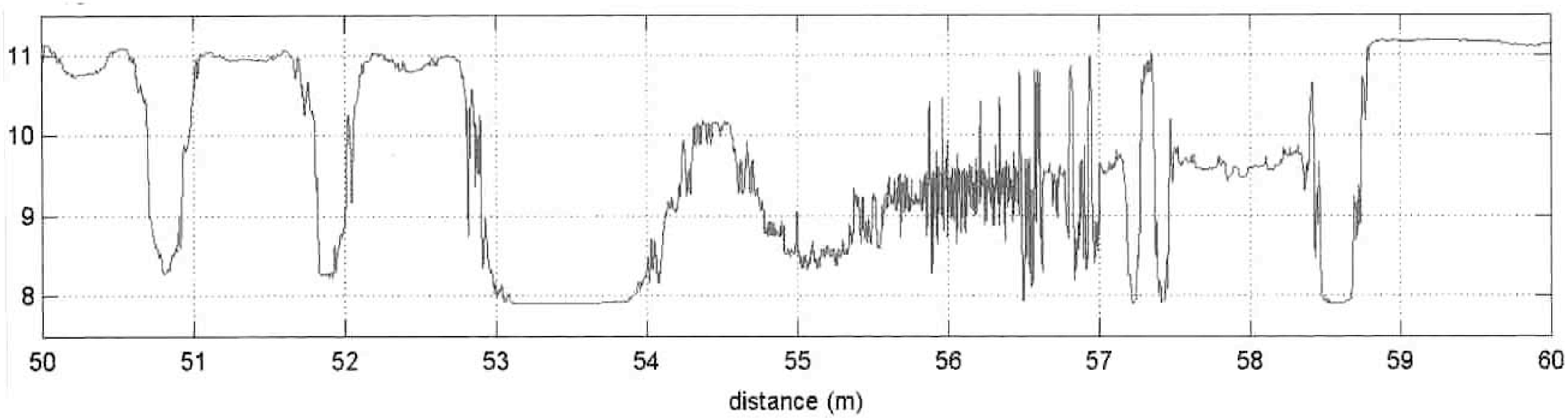
Figure 2.2. Effect of a single counterclockwise eddy on a scalar field with linear gradient.

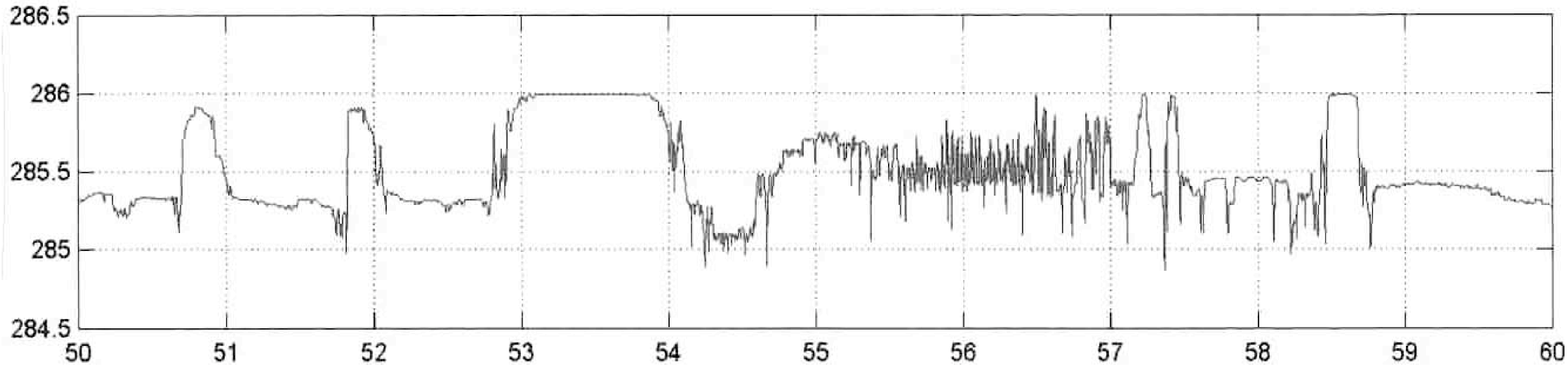
Triplet Map

Each triplet map has a location, size, and time.

- *Location* is randomly chosen.
- *Size* is randomly chosen from a distribution that matches inertial range scalings.
  - *Smallest* map (eddy) is Kolmogorov scale.
  - *Largest* eddy is L, usually domain size.
- Eddies occur at a *rate* determined by the large eddy time scale and eddy size range.

1-D Explicit Mixing Model: water vapor and temperature fields





1-D SGS model for LES

- *LES:* grid size = 10 m.
- *SGS mixing:* Explicit mixing model (EMM) with grid size =1 m.
- *Microphysics:* bin model in each EMM grid cell.
- *Collection kernel:* increased to represent turbulence effects.
- *CPU time* ~ LES with bin model at 5-m grid size.

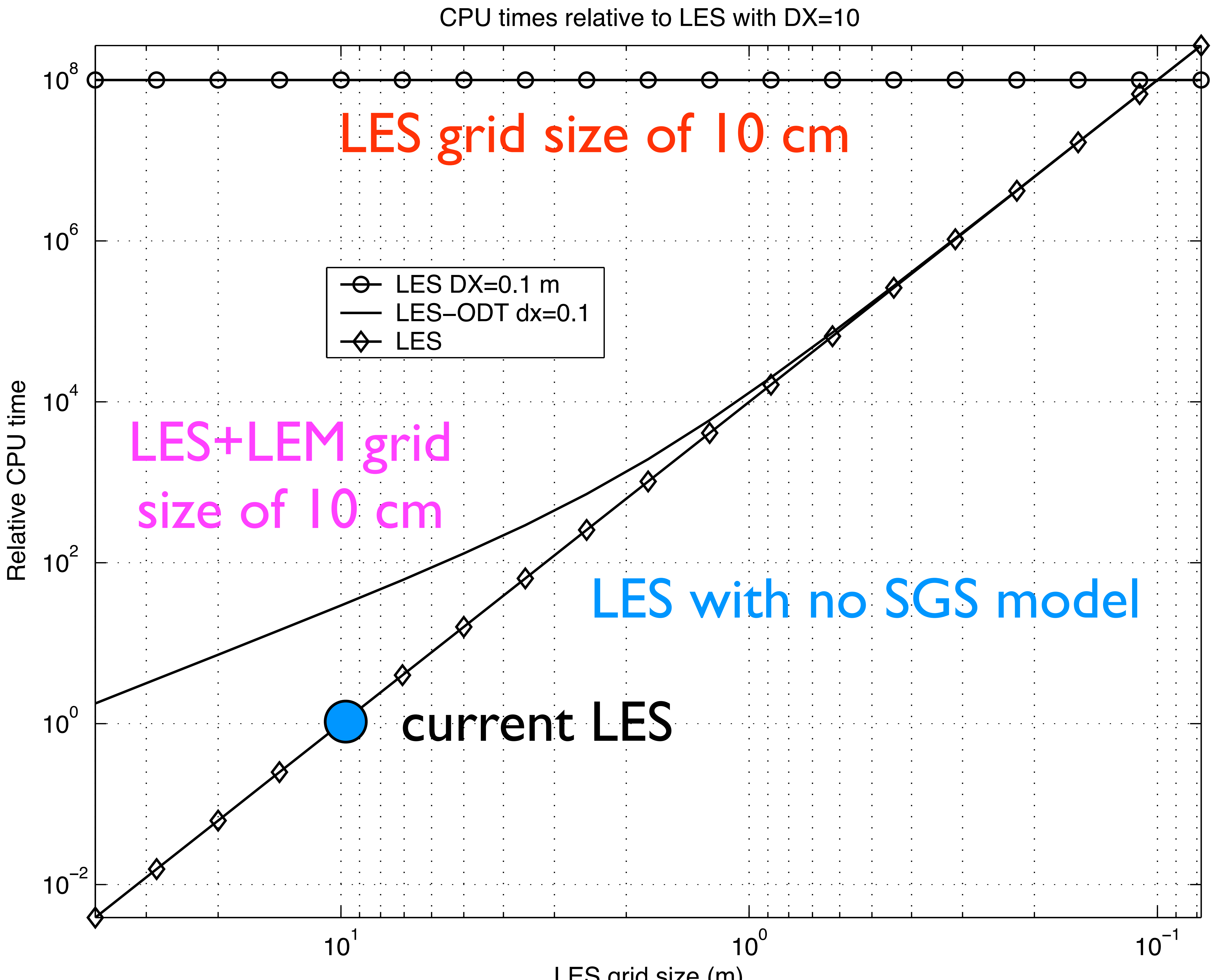
Summary

- Reducing the dimensionality is an established method.
- Removes or reduces the need for SGS parameterizations.
- It is very well suited for high-Reynolds number turbulent flows when small-scale mixing processes are important.

EXTRA SLIDES

1-D SGS models for LES

Turbulence	Physics	Microphysics	Dimension	Domain	Grid size	Grid points (Droplets)
LEM	mixing & buoyancy	bulk condensation & evaporation	1	10 m	1 m	10 -
LEM	mixing & DSD	droplet condensation & evaporation	1	10 m	1 mm	10^4 (10^3)
3D droplet triplet map	turbulence & collisions	droplet collision & coalescence	1 (3)	10 m $\times$ (1 cm)^2	-	- (10^3)
LEM	all	bin model: droplet cond/evap & coll/coal	1	10 m	1 cm	10^2 -


CPU times relative to LES with DX=10
LES grid size of 10 cm
LES DX=0.1 m
LES−ODT dx=0.1
LES
LES+LEM grid size of 10 cm
LES with no SGS model
current LES
Relative CPU time
LES grid size (m)
10^8
10^6
10^4
10^2
10^0
10^−2
10^1
10^0
10^−1

A 3D triplet map for inertial droplets

The droplet trajectory model idealizes droplet response to continuum flow (dashed curves: notional continuum fluid streamline and droplet trajectory)

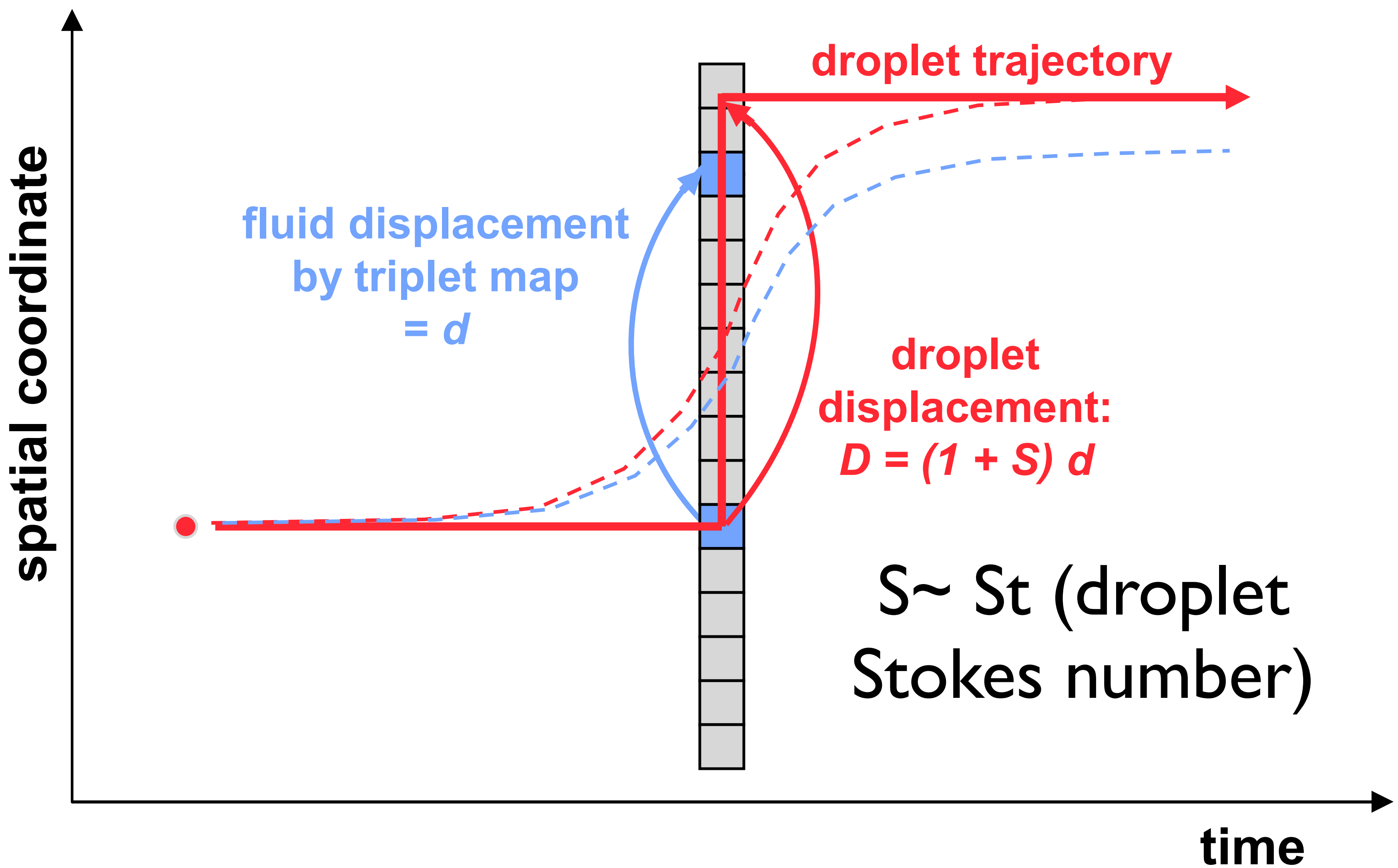
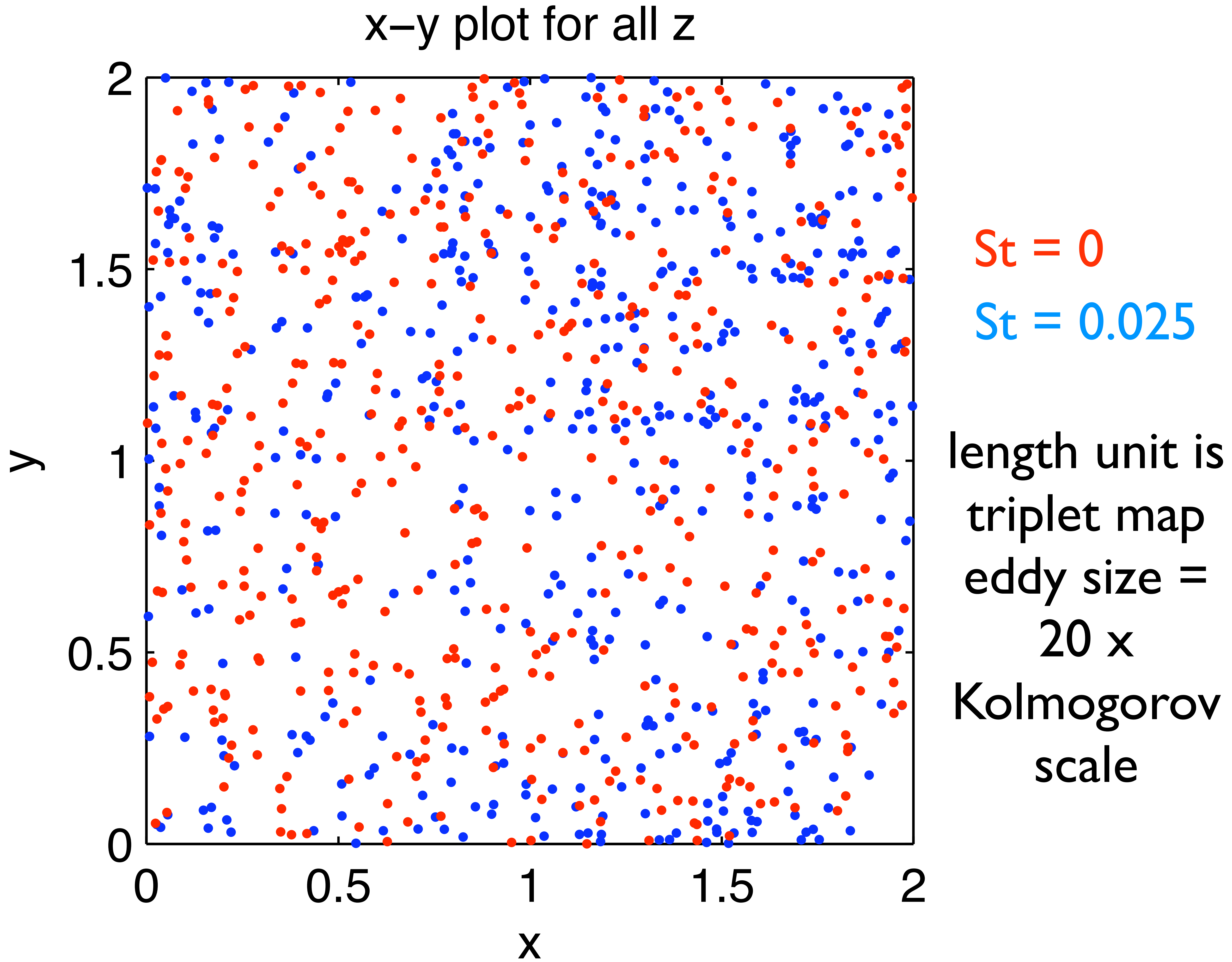


Fig 3. Difference between dispacement due to triplet map and the total displacement. (A. R. Kerstein)


x–y plot for all z
y
x
0
0.5
1
1.5
2
St = 0
St = 0.025
length unit is triplet map eddy size = 20 x Kolmogorov scale

Triplet map vs. 3D turbulence

Strengths

- Transport: map frequency is set so that fluid transport matches turbulent eddy diffusivity.
- Length scale reduction: by matching the inertial-range size-vs.-frequency distribution of eddy motions, the rate of length scale reduction as a function of fluid parcel size is consistent with 3D turbulence.
- Intermittency: Random sampling of triplet map occurrences and sizes reproduces, qualitatively and to some degree quantitatively, intermittency properties of 3D turbulence.
- Mixing: In conjunction with molecular diffusion, the map sequence reproduces mixing features.

Triplet map vs. 3D turbulence

Weaknesses

- Omits effects of time persistence of turbulent motions.
- When diffusive time scales are shorter than turbulent time scales, diffusion can suppress scalar fluctuations faster than they are generated in 3D turbulence.
- In some cases, turbulence spreads a slow-diffusing scalar faster than a fast-diffusing scalar. This is a multi-dimensional effect that 1D advection cannot capture.

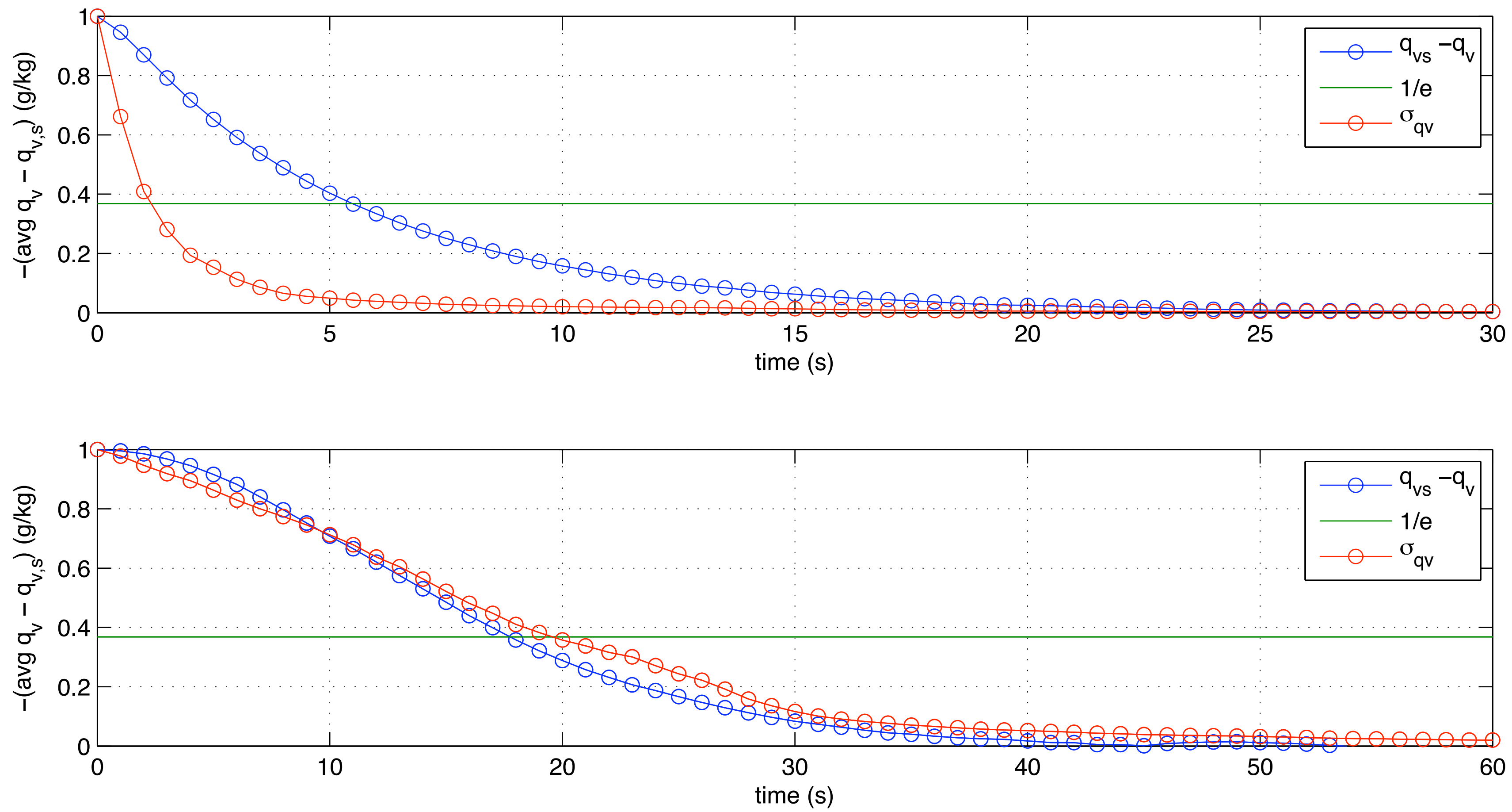


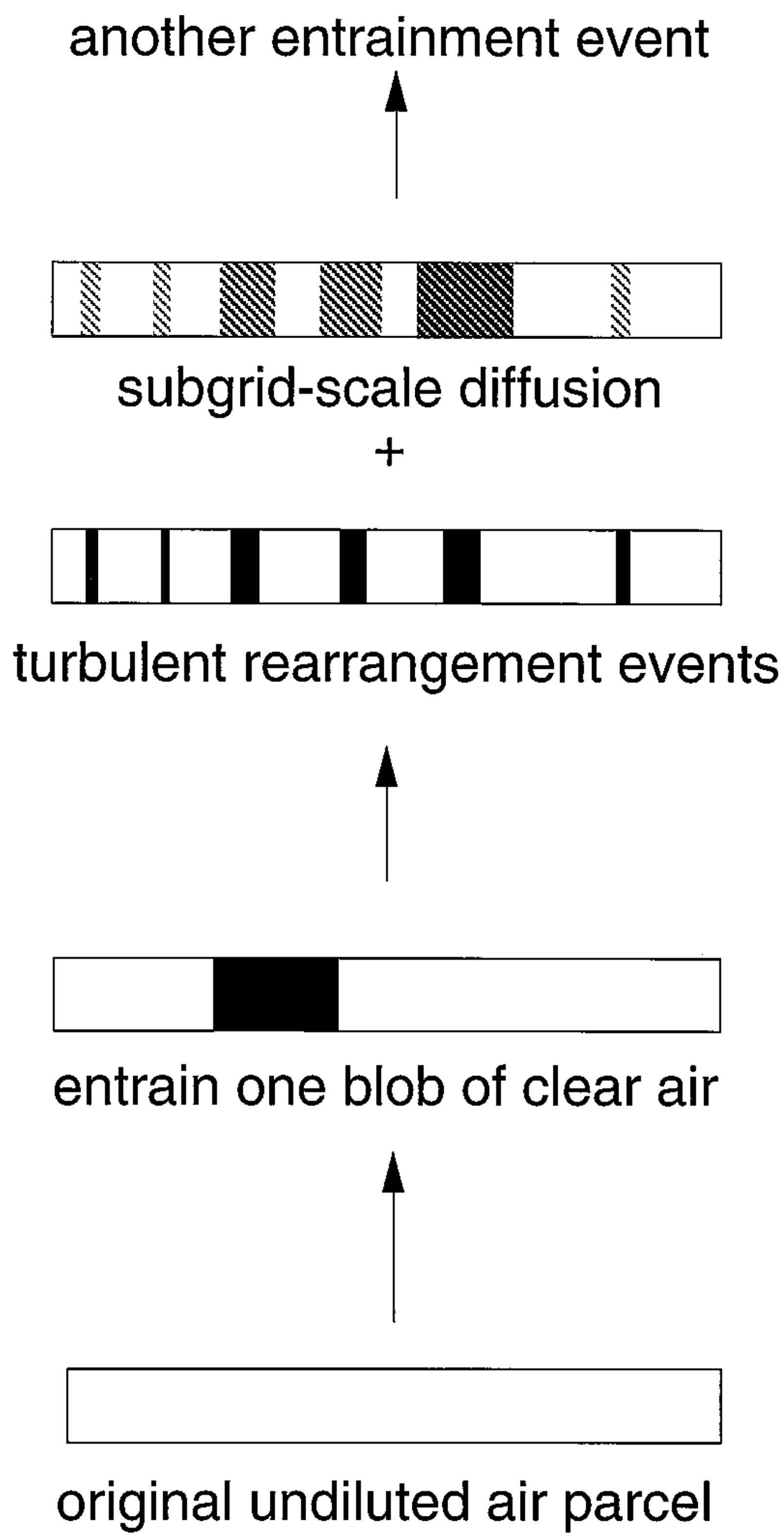
Figure 1: (top) Explicit Mixing Parcel Model simulation of isobaric mixing of saturated air (containing 100 cloud droplets per cm^{-3} of radius 15 μm) with 1 segment of subsaturated air 0.25 m in length in a 1D domain 20 m in length, with a dissipation rate of 10^{-2} m^2 s^{-3}. The blue curve is the average subsaturation normalized by its initial value, the red curve is the std dev of the water vapor mixing ratio, and the green line is $1/e$. The e-folding times for saturation adjustment and decay of water vapor std dev obtained from the plot are 5.5 s and 1.2 s, respectively. The calculated evaporation and mixing timescales are 4 s and 1.8 s, respectively. (bottom) Same as top except for mixing of 5 segments of subsaturated air (each 10 m in length) in a 1D domain 100 m in length. The e-folding times for saturation adjustment and decay of water vapor std dev obtained from the plot are 18 s and 20 s, respectively. The calculated evaporation and mixing timescales are 4 s and 22 s, respectively.

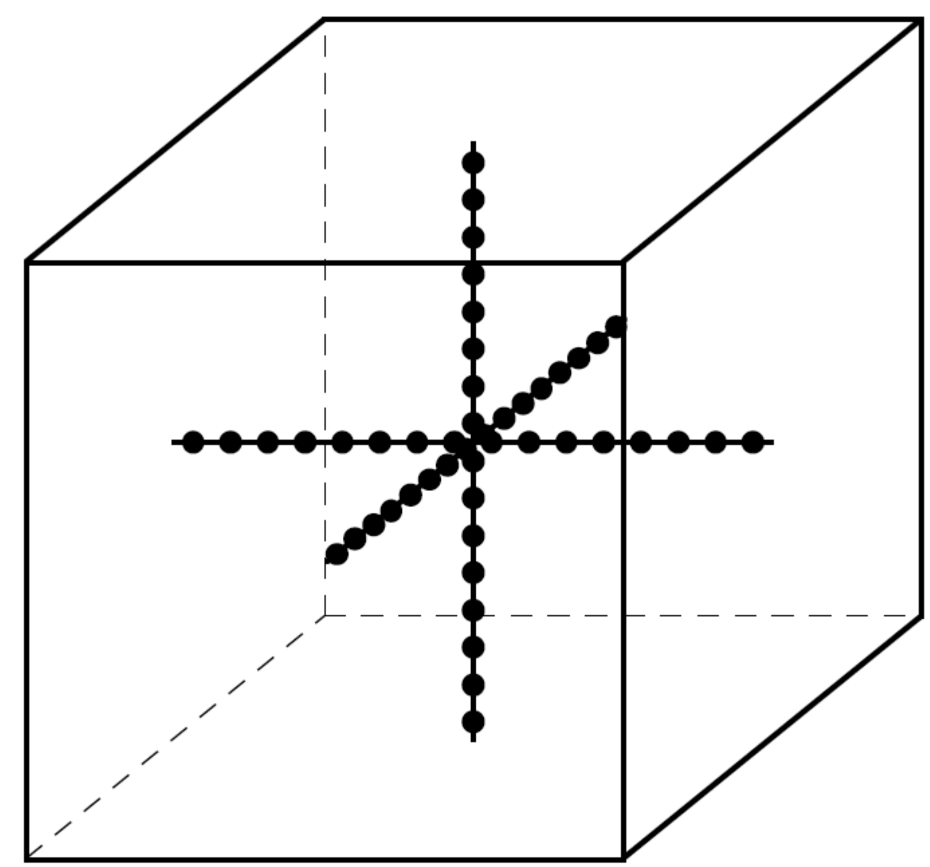
Parameterization of SGS Cloud Processes in LES

- SGS mixing is instantaneous in most LES.
- SGS variability does not affect DSD in any LES.
- SGS turbulence affects droplet collision rates in very few LES.

Explicit Mixing Parcel Model (EMPM)

LES with 1D subgrid-scale model





Explicit Mixing Parcel Model (EMPM)

- The EMPM predicts the evolving in-cloud variability due to entrainment and finite-rate turbulent mixing using a 1D representation of a rising cloudy parcel.

- The 1D formulation allows the model to resolve fine-scale variability down to the smallest turbulent scales ($\sim$ 1 mm).

- The EMPM can calculate the growth of several thousand individual cloud droplets based on each droplet's local environment.

EMPM Required Inputs

- Required for a classical (instant mixing) parcel model calculation:

  Thermodynamic properties of cloud-base air

  Updraft speed

  Entrainment rate

  Thermodynamic properties of entrained air

  Aerosol properties

- In addition, the EMPM requires:

  Parcel size

  Entrained blob size, d

  Turbulence intensity (e.g., dissipation rate, ϵ)

The droplet Stokes number is

$$\mathsf{St} = t_d \gamma$$

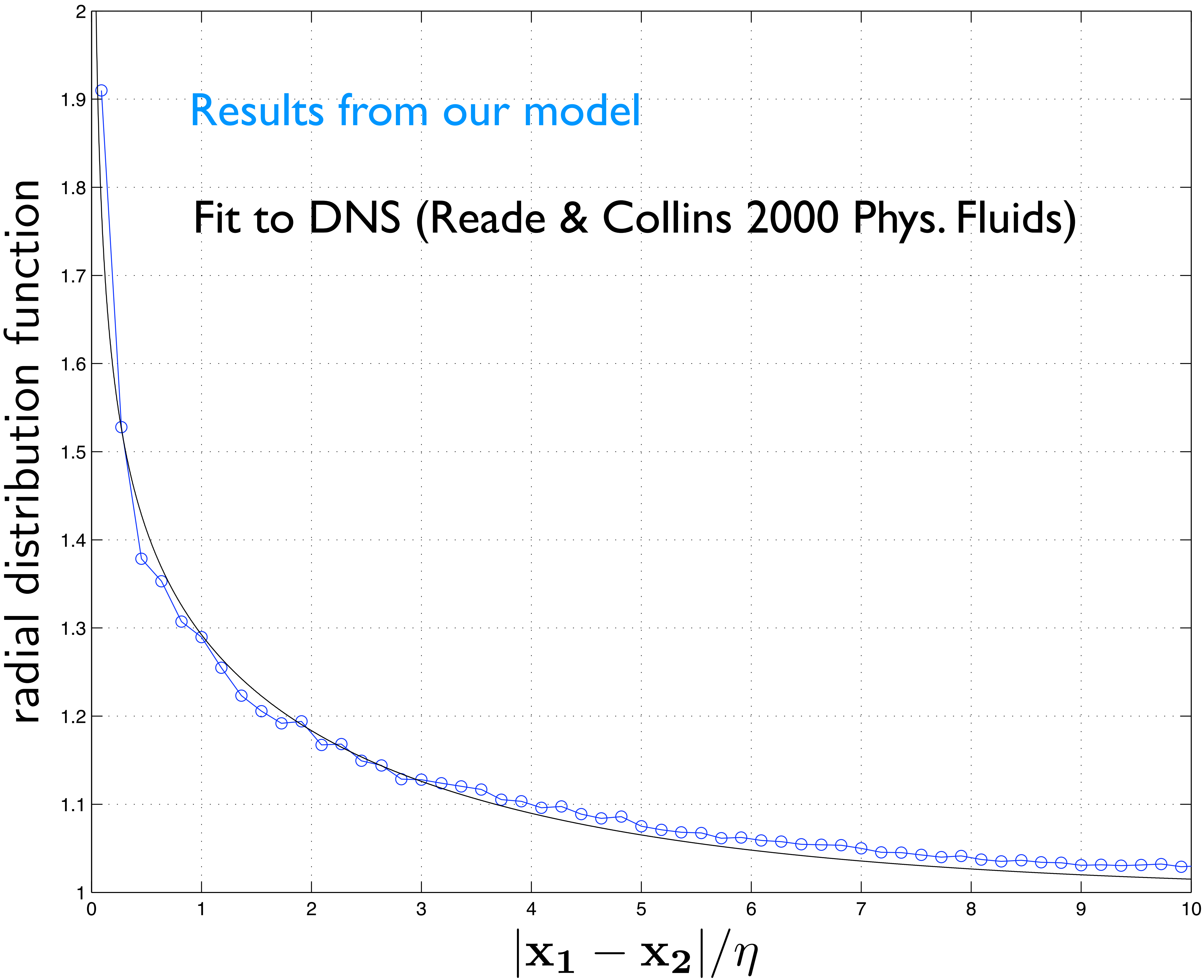
where

$$t_d = \frac{m_p}{6\pi r \mu} = \frac{2\rho_p r^2}{9\mu}$$

is the droplet response time, m_p is the droplet mass, ρ_p the droplet density, r the droplet radius, and μ the dynamic viscosity, and

$$\gamma = (\epsilon/\nu)^{1/2} = 1/\tau_K$$

is a global measure of strain, and τ_K is the Kolmogorov time scale.


Results from our model
Fit to DNS (Reade & Collins 2000 Phys. Fluids)
radial distribution function
$|\mathbf{x_1} - \mathbf{x_2}|/\eta$
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
1
2
3
4
5
6
7
8
9
10