Kelvin Waves and the Vertical Profile of Cumulus Entrainment

Walter Hannah and Eric Maloney
Colorado State University

Introduction

Several studies have found evidence from high resolution cloud resolving models that entrainment may be appropriately characterized as having the largest values near cloud base. Cumulus parameterizations based on these observations have shown notable improvements (Chikira and Sugiyama 2010; Murato and Ueno 2005; Sahany et al. 2012). Here we investigate this idea further using a relatively simple cumulus scheme in a coarse resolution GCM in the context of convectively coupled equatorial waves. Our central question then is:

How does enhanced low-level entrainment affect convectively coupled Kelvin waves in a GCM?

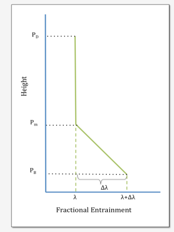

Fig 1. Schematic illustration of the entrainment modification.

Simulation	Min. Entrianment	Δλ [km^{-1}]	P_m [hPa]
Control	0.1	0	-
Tok	0.1 x 2	0	-
DeepA	0.1	0.8	~700
DeepA+Tok	0.1 x 2	0.8	~700
DeepB	0.1	1.0	~500

Model Setup

- NCAR CESM (Atmosphere only)
- T42 resolution
- Relaxed Arakawa-Schubert (RAS) cumulus scheme *(Moorthi and Suarez 1992)*
- UW shallow convection *(Park and Bretherton 2009)*

RAS was modified so that cumulus entrainment is a piece-wise linear function in height, with no major changes to the formulation of the scheme (see Fig. 1 & 2). By setting $\Delta\lambda=0$ we remove the low-level enhancement and revert back to the original RAS, which is used for our control simulation. The Tokioka et al. (1988) method of increasing the minimum entrainment is used for comparison, since it effectively enhances entrainment at all levels equally.

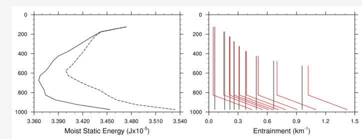

Fig 2. Example calculation of entrainment profiles (right) using an average sounding during convectively active periods from TOGA-COARE (left) with and without low-level enhancement (black and red lines respectively).

Tropical Precipitation Climatology

- Isolated areas of strong precipitation biases over land masses are present in the control (Fig. 3a)
- TRMM reveals much smoother gradients between areas of light and heavy rain (Fig. 3f)
- Both entrainment modifications result in an improved precipitation climatology, although still with some biases, such as a overly weak precipitation in the ITCZ and Indian Ocean and overly strong precipitation in the Northeast Pacific (Fig. 3b-e)

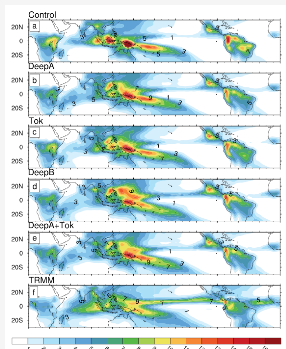

Fig 3. Annual mean precipitation in *mm day⁻¹*.

Precipitation Spectra

- Kelvin wave variance has too much power at large equivalent depths (*~200m*) suggesting that some convectively coupled waves propagate up to *~30m/s* faster in the *Control* and *Tok* simulations compared to TRMM observations (Fig. 4a,c)
- Enhanced low-level entrainment leads to less variance at large equivalent depths (Fig. 4b,d,e)

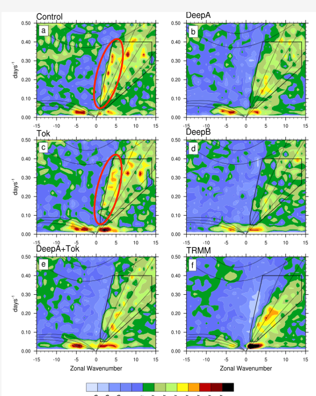

Fig 4. Spectral power normalized by an approximate background spectrum (i.e. signal-to-noise ratio estimate) for symmetric precipitation anomalies averaged from *10°S-10°N*. Dispersion curves derived from shallow water theory are shown for *n=1* Rossby, Kelvin and inertia-gravity waves with equivalent depths of *h=12, 25 and 50* m. The polygon shows a typical frequency domain used for filtering spanning *h=8* to *h=200*.

Kelvin Wave Regression

- Kelvin wave precipitation regression analysis reveals that a typical Kelvin wave propagates faster than observations in all simulations
- Enhanced low-level entrainment is more effective at reducing the fast Kelvin wave bias (Fig. 5b,d) compared to the Tokioka method (Fig. 5c,e).

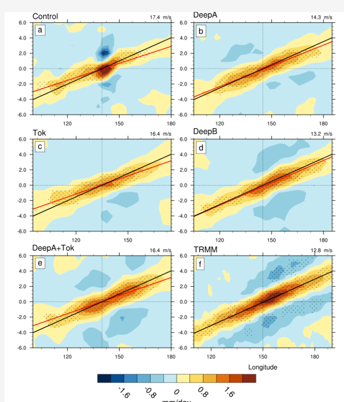

Fig 5. Lagged regression of equatorial precipitation (*10°S-10°N)* onto a Kelvin wave index at the point of maximum Kelvin wave variance indicated by the vertical line. The black line indicates a least squares estimate of the propagation speed estimated from TRMM and the red lines are that for each simulation. Stippling indicates significance at the 95% level.

Kelvin Wave Propagation

Using shallow water theory we can relate the propagation speed of a gravity wave to the structure of an assumed sinusoidal heating profile with vertical wavenumber *m* and buoyancy frequency *N* as

$$c^2 = gh_e = \frac{N^2}{m^2} = \frac{N^2H^2}{n^2\pi^2} \qquad (1)$$

$$\Delta c = c_2 - c_1 = \frac{H}{\pi}\left(\frac{N_2}{n_2} - \frac{N_1}{n_1}\right) \qquad (2)$$

From (2) we can estimate the expected change in Kelvin wave phase speed due to changes of *N* or *m* that we see in the model in Table 1.

H [km]	N1 [s^{-1}]	N2 [s^{-1}]	n1	n2	|Δc| [$m\ s^{-1}$]
20	0.01	0.011	1	-	6.4
20	0.01	-	1	2	31.8

Table 1. Estimated phase speed changes from eq. 2 due to select stability and vertical structure parameters to illustrate the sensitivity to typical values.

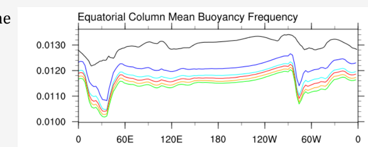

Fig 6. Annual, equatorial and column average buoyancy frequency. Calculated using potential temperature as $N^2 = (\theta/g)*(d\theta/dz)$.

- Both entrainment methods reduce *N* (Fig. 6), but the magnitude of the change is too small to explain the difference in equivalent depth (Kiladis et al. 2009).
- An increase in the contribution from higher vertical wavenumbers (Fig. 5) appears to be a more effective means for reducing Kelvin wave speed (Mapes 2000).

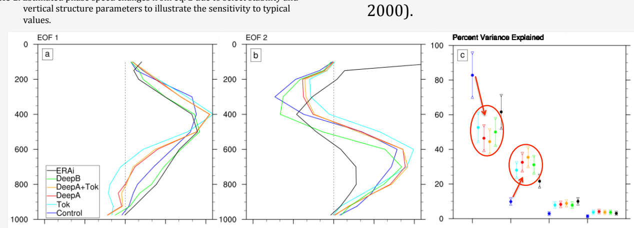

Fig 7. The left and center panels show the first and second EOF modes of the vertical profile of diabatic heating (i.e. Q1) associated with Kelvin waves in the Indo-Pacific region, respectively. The right panel shows the percent variance explain of the first four modes.

Summary

- Enhancing low-level entrainment does not affect annual mean tropical precipitation significantly different from the Tokioka method (Fig. 3)
- Enhanced low-level entrainment suppresses convectively coupled Kelvin waves that are too fast compared to observations, unlike the Tokioka method (Fig. 3)
- Reduced buoyancy frequency (i.e. stability; Fig. 6) of *~0.001* s^{-1} corresponds to Δc = ~6 $m\ s^{-1}$, which is too small considering the changes shown in Figure 4.
- Higher vertical wavenumbers (Fig. 7) can potentially explain a phase speed reduction of *~20-30* $m\ s^{-1}$, but the Tokioka results and ERAi data seem to be inconsistent with this idea.

We conclude that it is necessary to consider changes in both the buoyancy frequency and vertical heating structure to explain why enhanced low-level entrainment is more effective at reducing the fast Kelvin wave bias in the control. Compared to other studies, this suggests that including other aspects of convection such as downdrafts or convective momentum transport are more important than the vertical structure of the convective entrainment.

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