Introduction
Several studies have found evidence from high resolution cloud resolving models that entrainment may be inappropriate characterized as having the largest values near cloud base. Cumulus parameterizations based on these observations have shown notable improvements (Chikira and Sugiyama 2016; Murato and Ieno 2005; Sahany et al. 2012). Here we investigate this idea further using a relatively simple cumulus scheme in a coarse resolution GCM in the context of convectively coupled equatorial waves. Our central question then is:

How does enhanced low-level entrainment affect convectively coupled Kelvin waves in a GCM?

Tropical Precipitation Climatology
- Isolated areas of strong precipitation biases over land masses are present in the control (Fig. 3a)
- TRMM reveals much smoother gradients between areas of light and heavy rain (Fig. 3f)
- Both entrainment modifications result in an improved precipitation climatology, although still with some biases, such as a overly weak precipitation in the ITCA and Indian Ocean and overly strong precipitation in the Northeast Pacific (Fig. 3b-e)

Precipitation Spectra
- Kelvin wave variance has too much power at large equivalent depths (~200m) suggesting that some convectively coupled waves propagate up to ~30m/s faster in the Control and Tokioka simulations compared to TRMM observations (Fig. 4a,c)
- Enhanced low-level entrainment leads to less variance at large equivalent depths (Fig. 4d,e)

Kelvin Wave Regression
- Kelvin wave precipitation regression analysis reveals that a typical Kelvin wave propagates faster than observations in all simulations
- Enhanced low-level entrainment is more effective at reducing the fast Kelvin wave bias (Fig. 5d,e) compared to the Tokioka method (Fig. 5c,e).

Kelvin Wave Propagation
Using shallow water theory we can relate the propagation speed of a gravity wave to the structure of an assumed sinusoidal heating profile with vertical wavenumber $m$ and buoyancy frequency $N$ as

$$c = \sqrt{gN/m} \left( \frac{N^2}{m^2} + \frac{H^2}{m^2} \right)$$

From (2) we can estimate the expected change in Kelvin wave phase speed due to changes in $N$ or $m$ that we see in the model in Table 1.

<table>
<thead>
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<th>Simulation</th>
<th>Entrainment</th>
<th>$N$</th>
<th>$m$</th>
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<tbody>
<tr>
<td>Control</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Tokioka</td>
<td>0.1</td>
<td>-</td>
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<tr>
<td>DeepA</td>
<td>0.1</td>
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<td>DeepA+Tok</td>
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Table 1. Estimated phase speed changes from Eq. 2 due to reduced stability and convective activity parameterization in Kelvin wave theory.

- Both entrainment methods reduce $N$ (Fig. 6), but the magnitude of the change is too small to explain the difference in equivalent depth (Kanazawa et al. 2009).
- An increase in the contribution from higher vertical wavenumbers (Fig. 5) appears to be a more effective means for reducing Kelvin wave speed (Mapes 2000).

Summary
- Enhancing low-level entrainment does not affect annual mean tropical precipitation significantly different from the Tokioka method (Fig. 3)
- Enhanced low-level entrainment suppresses convectively coupled Kelvin waves that are too fast compared to observations, unlike the Tokioka method (Fig. 3)
- Reduced buoyancy frequency (i.e. stability; Fig. 6) of $-0.001m/s^2$ corresponds to $d = -6 m/s^2$, which is too small considering the changes shown in Figure 4.
- Higher vertical wavenumbers (Fig. 7) can potentially explain a phase speed reduction of $-29.30 m/s^2$, but the Tokioka results and ERA-Interim data seem to be inconsistent with this idea.

We conclude that it is necessary to consider changes in both the buoyancy frequency and vertical heating structure to explain how enhanced low-level entrainment is more effective at reducing the fast Kelvin wave bias in the control. Compared to other studies, this suggests that including other aspects of convection such as downdrafts or convective momentum transport are more important than the vertical structure of the convective entrainment.