Testing New NAVGEM Orographic GWD Parameterization Using DEEPWAVE Data

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Source Parameterizations Based on Surface Drag for 3D Elliptical Obstacles

1. Fit subgridscale orographic elevations $h(x,y)$ to an effective idealized anisotropic 3D obstacle

$$\sigma_{xx}^2 = \frac{\partial h}{\partial x} \frac{\partial h}{\partial x} = \frac{\pi}{6} K_0^{3/2} \left[ K_U^{3/2} - K_L^{3/2} \right] (3a + c),$$

$$\sigma_{xy}^2 = \frac{\partial h}{\partial x} \frac{\partial h}{\partial y} = \frac{\pi}{3} K_0^{3/2} \left[ K_U^{3/2} - K_L^{3/2} \right] b,$$

$$\sigma_{yy}^2 = \frac{\partial h}{\partial y} \frac{\partial h}{\partial y} = \frac{\pi}{6} K_0^{3/2} \left[ K_U^{3/2} - K_L^{3/2} \right] (a + 3c),$$

Fr = $U/Nh_m < 1$

Fr$_c$ = $U/N(h_m - z_c) = 1$

2. Infer pressure “drag” $D_p$ from linear relations $D_L$

$$|D_L| = D_L = \frac{\pi}{4} a \rho_0 NU^2 h_m^2 = \frac{\pi}{4} a \rho_0 \frac{U^3}{N} Fr^{-2},$$

$$\tau_{sx} = \rho u N \hat{K}^{-1} (\sigma_{xx} \cos \chi + \sigma_{xy} \sin \chi),$$

$$\tau_{sy} = \rho v N \hat{K}^{-1} (\sigma_{xy} \cos \chi + \sigma_{yy} \sin \chi),$$

3. Split $D_p$ into wave ($D_w$) and surface ($D_p - D_w$) components, based on a dividing streamline $z_c$.

$$D_p = \int\int_{-\infty}^{+L/2} p' \left( \frac{\partial h}{\partial x}, \frac{\partial h}{\partial y} \right) dx dy,$$

$$D_w(z) = \rho(z) \int\int_{-L/2}^{+L/2} (u'w', v'w') dx dy.$$
3D OMD Regime Diagram

Elliptical Three-Dimensional Hill

\[ h(x, y) = \frac{h_m}{[1 + (x/a)^2 + (y/b)^2]^{3/2}} \]

Constant Upstream Flow Profile

\[ U = 10 \text{ m s}^{-1} \quad N = 0.01 \text{ rad s}^{-1} \]

Normalized Hill Height (inverse Froude number)

\[ \hat{h}_m = Nh_m/U = Fr^{-1} \]

Obstacle Aspect Ratio \[ \beta = \frac{b}{a} \]

Cases Simulated Using Mesoscale Model
3D OMD Regime Diagram

Elliptical Three-Dimensional Hill
\[ h(x, y) = \frac{h_m}{\left[1 + (x/a)^2 + (y/b)^2\right]^{3/4}} \]

Constant Upstream Flow Profile
\[ U = 10 \text{ m s}^{-1} \quad N = 0.01 \text{ rad s}^{-1} \]

Normalized Hill Height (inverse Froude number)
\[ \hat{h}_m = Nh_m / U = Fr^{-1} \]

Obstacle Aspect Ratio \( \beta = \frac{b}{a} \)

Horizontal Geometrical Spreading Effects on Wave Amplitude
Horizontal Geometrical Spreading

\[ \frac{\partial A}{\partial t} + \nabla \cdot (c_g A) = 0. \]

\( A = \) wave action density = \( E/\omega \)
\( E = \) total wave energy density (KE + PE)
\( \omega = \) wave intrinsic frequency
\( c_g = \) vector group velocity

\[ c_{gz} A J_h = \text{constant} \]

\[ J_h = \frac{\partial (x, y)}{\partial (x_0, y_0)} \]

The Jacobian \( J_h \) tracks change in horizontal cross-sectional area \( a(z) \) of a “ray tube”

\[ c_{gz} A(0) a(0) \approx c_{gz} A(z) a(z) : \]

\( a(z)/a(0) \gg 1 \rightarrow A(z)/A(0) \ll 1 \)

1. Wave breaking is a local criterion:
   \( A(x,y,z) \geq A_{\text{break}} \).

2. Parameterizations all assume no spreading:
   \( a(z)/a(0)=1 \quad J_h=1 \)
Geometrical Spreading Theory


Adding Geometrical Spreading Terms of OGWD Parameterizations

\[
\eta_{a_{\text{new}}}(z) = a_{\eta}(z)\eta_a(z) = \eta_a(0)a_{\eta}(z) \left[ \frac{m(z)\rho(0)N^2(0)}{m(0)\rho(z)N^2(z)} \right]^{1/2}.
\]

Small-\(l\) Approximation  | \(l/k\) | << 1

\[
\hat{a}_{\eta_{\text{si}}} (z') = \frac{\hat{a}_{\eta_{\text{si}}}(0,0,z')}{h_m G_\eta(z)} = \exp \left[ i z' \left( \frac{1}{2\beta^2} - 1 \right) \right] \text{erfc} \left( \frac{z'}{\beta^2} \right)^{1/2} \frac{(1+i)}{2}.
\]

Single-\(k\) Approximation  \(k = 1/\gamma a\)

\[
\hat{a}_{\eta_{\text{sk}}} (z') = \frac{\hat{\eta}_{\text{sk}}(0,0,z')}{h_m G_\eta(z)} = \frac{e^{-iz'}}{(2iz'\gamma^2/\beta^2 + 1)^{1/2}}.
\]

\[
z' = \int_0^z \left[ \frac{N(\tilde{z})}{|U(\tilde{z})|} \right] d\tilde{z},
\]

Let \(z'/\beta^2 >> 1\)

\[
|\hat{a}_{\eta_{\text{sk}}} (z')| \simeq |\hat{a}_{\eta_{\text{sk}}} (z')| \simeq \left( \frac{2}{\pi} \right)^{1/2} \left( \frac{z'}{\beta^2} \right)^{-1/2}
\]

\(U = 10\) ms\(^{-1}\), \(N = 0.01\) s\(^{-1}\)

- \(\beta = 1/10, z'/\beta^2 = 1 \Rightarrow z = \beta^2 U/N = 10\) meters
- \(\beta = 10, z'/\beta^2 = 1 \Rightarrow z = \beta^2 U/N = 100\) km
Horizontal Geometrical Spreading Curves

Hydrostatic Steady State, \( U = 10 \text{ m/s}, N = 0.01 \)

\[
h(x, y) = \frac{h_m}{\left[1 + (x/a)^2 + (y/b)^2\right]^p},
\]

\[
\beta = \frac{b}{a}
\]
Sensitivity to Mountain Shape & Wind

\[ z' = \int_0^z \left[ \frac{N(\tilde{z})}{|U(\tilde{z})|} \right] d\tilde{z}, \]

3D Orography and Projections
Relevance to Wave Breaking Parameterization

Peak Amplitude: $\rho^{-1/2}$ Density Scaling

Current "No Spreading" Approximation Used in all Existing GWD Parameterizations
Test these Ideas for DEEPWAVE
Auckland Island Case

1. Use 0-100 km winds and temperatures from NAVGEM reanalysis to define background
   a. Note: surface Froude numbers $Fr_0 = |U_0|/N_0 h_0 \geq 3$
2. Derive exact 3D linear transform solutions using Auckland Island topography and $U(z)$, $V(z)$ and $T(z)$ profiles from (1)
3. Compare to RF23 AMTM images
4. Compare to orographic gravity wave drag parameterizations
Tests: Leverage Linear RF23 Solutions

No wavebreaking until $\sim 90$ km altitude.
Wave activity gets to 90 km rapidly! (2-4 hours)
At lower altitudes, filtering by turning points and critical layers helps keep wave amplitudes small.
Comparison of RF23 OGWs to Parameterization Approximations

Vertical Displacement $\eta$
- Old Parameterization
- New Parameterization
- Exact 3D Solution

Steepness $\eta_z$
- Old Parameterization
- New Parameterization
- Exact 3D Solution
Summary So Far......

- Validating/improving OGWD parameterizations is a major impetus for Navy involvement in DEEPWAVE.
- Specific DEEPWAVE OGW cases are already providing a very useful environment for objectively testing new features developed for the NAVGEM parameterization of subgridscale OGWD.
- Early work, need more guidance from 3D modelers and measurement teams.
Locations of Wave Amplitude Maxima Remain Surprisingly Close to the Mountain.

Vertical Displacement Maximum Amplitude

(a) Local Maxima for $\beta=1$

(b) Local Maxima for $\beta=1/2$

(c) Local Maxima for $\beta=1/4$

(d) Local Maxima for $\beta=1/8$
Gaussian Elliptical Orography

The Fourier transform of this Gaussian hill function is

\[
\hat{h}(k, l) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) e^{-i(kx+ly)} \, dx \, dy,
\]

\[
= \frac{h_{m}ab}{4\pi} \exp \left( -\frac{k^2 a^2}{4} - \frac{l^2 b^2}{4} \right).
\]

Hydrostatic Dispersion Relation

where

\[
\bar{\eta}(0, 0, z') = \frac{h_{m}ab}{2\pi} G_{\eta}(z) \int_{-\infty}^{\infty} \int_{-\infty}^{0} e^{i(k,l,z')} \, dk \, dl,
\]

3D Mountain Wave Transform Solution

Above Mountain at \( x=y=0 \)

How to approximate the \( \exp(\chi) \) integral?

Small-\( l \) Approximation \( | l/k | << 1 \)

\[
\left( 1 + \frac{l^2}{k^2} \right)^{1/2} \approx 1 + \frac{l^2}{2k^2}.
\]

The small-\( l \) approximation of (16) is then

\[
\chi_{si}(k, l, z') \approx -iz' \left( 1 + \frac{l^2}{2k^2} \right) - \frac{k^2 a^2}{4} - \frac{l^2 b^2}{4}.
\]

Single-\( k \) Approximation \( k = 1/\gamma a \)

\[
(1 + l^2/k^2)^{1/2} \approx 1 + \frac{1}{2} l^2 \gamma^2 a^2,
\]

leading to a single-\( k \) approximation to the exponential argument (16) of

\[
\chi_{sk}(k, l, z') \approx -iz' \left( 1 + \frac{l^2 \gamma^2 a^2}{2} \right) - \frac{k^2 a^2}{4} - \frac{l^2 b^2}{4},
\]

\[
= -iz' - \frac{k^2 a^2}{4} - \frac{l^2 b^2}{4} \left( 2z' \gamma^2 / \beta^2 + 1 \right).
\]

\[
\hat{\eta}_{sk}(z') = \frac{\hat{\eta}_{sk}(0, 0, z')}{h_{m} G_{\eta}(z)} = e^{-iz'} \left( \frac{2z' \gamma^2 / \beta^2 + 1}{2z' \gamma^2 / \beta^2 + 1} \right)^{1/2}.
\]
Analytical Approximations

(a) $|\hat{\eta}(z^*)| \beta=0.125$

(b) $|\hat{\eta}(z^*)| \beta=0.2$

(c) $|\hat{\eta}(z^*)| \beta=0.333$

(d) $|\hat{\eta}(z^*)| \beta=0.5$

(e) $|\hat{\eta}(z^*)| \beta=1$

(f) $|\hat{\eta}(z^*)| \beta=2$

(g) $|\hat{\eta}(z^*)| \beta=3$

(h) $|\hat{\eta}(z^*)| \beta=5$

(i) $|\hat{\eta}(z^*)| \beta=8$
Momentum Fluxes at 85 km

\[ \rho < uw > \]

\[ \rho < vw > \]
Parameterization of 3D OMD Regime

**Deterministic Dividing Streamline**
1. Flow-Blocking Drag
   - Deterministic Magnitude
   - **Stochastic Direction**
2. Gravity-Wave Flux
   - Deterministic magnitude
   - Deterministic direction (parallel to incident flow)
   - **Geometrical Spreading**

**Deterministic Pressure Drag**
1. 100% 3D Gravity Wave Flux
   - Deterministic Magnitude
   - Deterministic Direction
   - **Geometrical Spreading**
Quantifying Horizontal Geometrical Spreading Effects on Wave Amplitude
Eckermann et al., JAS, in press, 2015a 2015b

- Use a Hilbert transform technique to drive local wave amplitudes from exact numerical transform solutions for linear three-dimensional mountain waves
  \[ \tilde{X}(x, y, z) = X_A(x, y, z) e^{i\psi} x(x, y, z) \]
- Locate and quantify largest wave amplitude at each altitude (most likely location for wave breaking)
  \[ X_A^{max}(z) = \max [X_A(x, y, z)] \]
- For hydrostatic solutions, vertical refraction terms that affect wave amplitudes can be well approximated by simple height profiles \( G(z) \) that depend only on background atmospheric parameters: e.g.,
  - Hydrostatic WKB solutions have an \( [m(z)/(m(0))]^{1/2} \) amplitude dependence with height for vertical displacements \( \eta(x, y, z) \)
  - yet vertical wavenumbers \( m(z) \approx N(z)/U(z) \) where \( N(z) \) is buoyancy frequency and \( U(z) \) is horizontal wind profile, thus \( G(z) = [N(z)U(0)/N(0)U(z)]^{1/2} \).
- Normalize the peak wave amplitudes to isolate the horizontal geometrical spreading effect on wave amplitude evolution with height
  \[ a_\eta(z) = \frac{\eta_A^{max}(z)}{h_m G_\eta(z)} \]
Hilbert Transform Removal of Phase to Yield Peak Amplitude Solutions

(a) Zonal Vertical Cross Section

(b) Zonal Vertical Cross Section

(c) Horizontal Cross Section @5km

(d) Horizontal Cross Section @5km
Hilbert Transform Removal of Phase to Yield Peak Amplitude Solutions

\[ a_\eta(z) = \frac{\eta_A^{max}(z)}{h_m G_\eta(z)} \]

- \( z \sim 5 \text{ km} \)
- \( h_m = 100 \text{ m} \)
- \( G_\eta(z) = 1 \)
- \( \eta_{max} \sim 34 \text{ m} \)
- \( a_\eta(z) = 0.34 \)