Mt Cook, South Island, New Zealand

Broad Spectrum Mountain Waves

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Questions

• Why are the DEEPWAVE A/C mountain wave spectra so broad?
• Why are the u-power, MF and w-power spectra so different?
• Is flow over the NZ massif more important than flow into the valleys?
• What wavelength waves carry most of the momentum flux?
• Does WRF also give broad spectra?
• What are the implications of broad spectra?
Spectral Variances

- Mountain shape or streamline displacement $\eta(x)$
- Fourier Transform $\hat{\eta}(k) = \int_{-\infty}^{\infty} \eta(x) \exp(-ikx) \, dx$
- $Var(\eta) = \int_{-\infty}^{\infty} \eta^2(x) \, dx = \left(\frac{1}{2\pi}\right) \int_{-\infty}^{\infty} \hat{\eta}(k)\hat{\eta}(k)^* \, dk$
- From hydrostatic mountain wave theory
  - $Var(w) = \int_{-\infty}^{\infty} w^2(x) \, dx = \left(\frac{U^2}{2\pi}\right) \int_{-\infty}^{\infty} k^2 \hat{\eta}(k)\hat{\eta}(k)^* \, dk$
  - $Cov(u, w) = \int_{-\infty}^{\infty} u(x)w(x) \, dx = -\left(\frac{NU}{2\pi}\right) \int_{-\infty}^{\infty} |k| \hat{\eta}(k)\hat{\eta}(k)^* \, dk$
  - $Var(u) = \int_{-\infty}^{\infty} u^2(x) \, dx = \left(\frac{N^2}{2\pi}\right) \int_{-\infty}^{\infty} \hat{\eta}(k)\hat{\eta}(k)^* \, dk$
- Note: P-power and T-power are similar to u-power
Monochromatic Wave
(blue line: typical buoyancy cut-off)

ParabolaCosine (n=11, beta=1)

u-power

MF & EFz

w-power
Ideal Rough hill
(blue line: typical buoyancy cut-off)

ParabolaCosine($n=11$, $\beta=0.3$)

(a)

ParabolaCosine($n=11$, $\beta=0.3$)

(b)

$10^9 K_0$ type spectrum

$10^4 K_1$ type spectrum

(c)

$10^4 K_2$ type spectrum

(d)

MF & EFz

u-power

w-power

define Volume and Roughness Modes
New Zealand transect, Mt. Cook
(blue line: typical buoyancy cut-off)

(a) $h(x)$
(b) $\times 10^9 K0$ type spectrum
(c) $\times 10^4 K1$ type spectrum
(d) $K2$ type spectrum

$u$-power

MF & EFz

w-power
Terrain with volume and roughness (hydrostatic results)

• Variance spectra are broad
• Volume mode dominates the $u$-power
• Roughness mode dominates the $w$-power
• Both modes contribute to MF and EFz
  – Volume mode: large $u'$ and small $w'$
  – Roughness mode: small $u'$ and large $w'$
Non-hydrostatic waves near the buoyancy cut-off

\[ k = \frac{N}{U} \]
\[ \lambda = \frac{2\pi U}{N} \approx 8 \text{km} \]
Non-hydrostatic waves near the buoyancy cut-off

• *Vertical wavenumber* \((m)\) approaches zero for \(k = N/U\)

\[
m = \left(\frac{N}{U}\right) \left[1 - \left(\frac{kU}{N}\right)^2\right]^{1/2}
\]

• *Deep vertical penetration*
• Wind perturbation approaches zero

• \(\hat{u} = \left(\frac{m}{k}\right) \hat{w}\)
• Momentum flux approaches zero
Non-hydrostatic waves near the buoyancy cut-off
(m=0; constant streamline spacing: u’=0)
Waves near the buoyancy cut-off
(good $w$ penetration, little $u'$ or MF)

Example at $z=12\text{km}$: roughness wavelength close to the buoyancy cut-off
Mountain Wave Flight

Ocean leg

RF09

Cross-mountain leg
Mt Aspiring

RF04: 7 legs

Vertical displacement

Mountain to scale but offset vertically

$EF_z = 2.8, 2.5, 5.1, 3.9, 2.8, 0.5, 3.5 \text{ W/m}^2$
RF05: 9 Legs

Vertical displacement

Mt Cook

EFz = 3.1, 1.7 W/m²

EFz = 5.0, 3.5, 3.6, 5.4, 5.3, 5.1, 6.7 W/m²

Mountain to scale but offset vertically
Mt Cook

RF09: 6 legs

Vertical displacement

Mountain to scale but offset vertically

EFz = 2.8, 2.1 W/m²

EFz = 3.7, 5.5, 7.3, 9.3 W/m²
Aircraft Transect Spectra

Blue line is the buoyancy cut-off
2km WRF Simulated Transect Spectra

Blue line is the buoyancy cut-off
Compare Variances from Volume and Roughness Modes

- WRF with 2km grid
- 3-day wave event from DEEPWAVE, July 3-5, 2014
- Use a high-pass and low-pass spectral filter
2km WRF Simulation: 3-day wave event; z=12km

w-power

MFx

u-power

Wind Speed & Direction

Red = Roughness Mode $\lambda < 60km$

Blue = Volume Mode $\lambda > 60km$

Black = Total
Spectral contributions

• Roughness Mode I: $\lambda = 8 \text{ to } 15 km$
  – Near the buoyancy cut-off
  – Non-hydrostatic
  – Dominates w-power. Very little u-power.
  – Little MF or EFz
• Roughness Mode II: $\lambda = 15 \text{ to } 40 km$
  – Large w-power
  – Small but significant u-power
  – Dominates MF and Efz
  – Nearly hydrostatic
• Volume Mode: $\lambda = 60 \text{ to } 300 km$
  – Large u-power
  – Small but significant w-power
  – Carries 1/3 or less of the MF and EFz
  – Dominates the P-power and T-power too.
  – Hydrostatic
Implications of Broad Spectra

• Aircraft sensors must have accurate broad-band response. (OK, I think)
• Numerical models must capture the Roughness Mode II: $\lambda = 15 \text{ to } 40km$ to get the MF and EFz right.
• Satellite IR sensors and Rayleigh Lidar will mostly see the T-power in the Volume Mode: $\lambda = 60 \text{ to } 300km$; missing the MF in Roughness II.
• Balloon data mostly see the u-power in the Volume Mode
• Usual monochromatic relations between u, w, T, p and MF do not apply.
The wave spectrum will influence wave breaking and GWD

- **Volume Mode** ($\lambda = 60 \text{ to } 300km$)
  - Dominates u-power, stagnation and wave breaking
  - Has steepening levels (not included in saturation theory)

- **Roughness Mode II** ($\lambda = 15 \text{ to } 40km$)
  - Dominates the MF and Efz
  - Weak u-power
  - MF deposition controlled by the Volume Mode

See talk by Chris Kruse.