Mountain Wave Attenuation in Positive and Negative Ambient Vertical Wind Shear

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NCAR’s Computational Resources (CISL, Yellowstone)
Background/Motivation

(a) Avg U (m s\(^{-1}\))

2014 Austral Winter

(c) Avg MF\(_x\) (mPa)

Height (km)

Kruse and Smith 2016
Background/Motivation

The Valley Layer restricts momentum flux analogous to a valve restricting mass flux through a pipe. Little mountain wave $MF_x$ gets through a weak wind.

Kruse and Smith 2016

Averaging Area

Averaging Area

2014 Austral Winter

Avg U (m s$^{-1}$)

Avg $MF_x$ (mPa)

Height (km)

Height (km)
Background/Motivation

2014 Austral Winter

Largest mid-stratospheric MF occurs when vertical wind shear is positive.

Kruse and Smith 2016
Questions

1. How does attenuation and resulting momentum deposition vary with altitude?

2. Does wind shear affect wave attenuation?

3. How do different scales interact during wave breaking?
Idealized Model Setup

- **Weather Research and Forecasting Model**
- **Setup (for now)**
  - 2-D
  - Horizontally Periodic
  - Constant $N = 0.02 \text{ s}^{-1}$
  - No Coriolis Force
  - Inviscid
- **3 Wind Profiles**
- **2 Compact Terrains**
Idealized Profiles guided by actual stratospheric profiles
Idealized Terrains

\[ h(x) = \begin{cases} 
\frac{h_m}{2} (1 + \cos(kx))(1 - \beta + \beta \cos(nkx)) & , |x| \leq d \\
0 & , |x| > d 
\end{cases} \]

- \( h_m = 500 \) m: max terrain height
- \( k = \pi/d \) where \( d \) is the range half-width
- \( \beta \) controls valley depth
- \( n = 7 \): number of smaller scale peaks

\[ \lambda_x = 200 \text{ km} \]
\[ \lambda_x = 200, 28.5 \text{ km} \]

Actual New Zealand Transect
Outline

1. Linear Theory Predictions

2. Levels of Wave Overturning/Attenuation
   – Many? Periodic?

3. Attenuation Diagnostic

4. Scale Interaction
1. WKB Predicted Overturning Altitude ($z_b$)

- Wave reaches overturning amplitude when u-amplitude equal to ambient wind
  - i.e. wave causes stagnation

- WKB Non-Linearity Ratio

$$NLR(z) = \frac{\hat{u}(z)}{U(z)}$$

$$NLR(z) = NLR_0 \left( \frac{U_0}{U(z)} \right)^{3/2} e^{z/2H_p}$$

$$NLR_0 = \frac{Nh_m}{2U_0} = \frac{1}{6}$$

$h_m=500$ m, $U_0=30$ m s$^{-1}$, $N=0.02$ s$^{-1}$

- $NLR(z_b) = 1$

<table>
<thead>
<tr>
<th>$Z_b$</th>
<th>No Shear</th>
<th>Positive Shear</th>
<th>Negative Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25.1 km</td>
<td>35.3 km</td>
<td>15.3 km</td>
</tr>
</tbody>
</table>
1. WKB Predicted Time to Overturning ($\tau_b$)

- Steady, Hydrostatic vertical group velocity:

  \[
  c_{g_z} = \frac{U^2 k}{N}
  \]

- Time to propagate to overturning altitude:

  \[
  \tau_b = \int_{0}^{z_b} \frac{1}{c_{g_z}} \, dz
  \]

<table>
<thead>
<tr>
<th>Mode</th>
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<th>Negative Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>200 km</td>
<td>296 min</td>
<td>292 min</td>
<td>218 min</td>
</tr>
<tr>
<td>Roughness</td>
<td>28.5 km</td>
<td>42 min</td>
<td>42 min</td>
<td>31 min</td>
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1. Wave Field at time=$\tau_b$

- Wave gets to $z_b$, but no wave overturning

Color Shading: Zonal Wind
Contours: Isentropes
1. Wave Field at time $= 2\tau_b$

- Wave overturning occurs near $t = 2\tau_b$
1. Wave Field at time=2τ_b

- Wave overturning occurs near t = 2τ_b

No Shear: Many overturning levels
Positive Shear: Fewer, spaced overturning levels
Negative Shear: Overturning confined to negative shear layer

Color Shading: Zonal Wind
Contours: Isentropes
2. Attenuation Isolated in the Vertical?

- Overturning periodic in vertical for compact, smooth Volume Mode
  - Momentum deposition also periodic!

\[ t = 2\tau_b \text{ for Volume Mode} \]
2. Attenuation Isolated in the Vertical?

- Overturning smooth in vertical for multiple mountain terrains
  - Initial momentum deposition also smooth

\[ t = 2\tau_b \text{ for Roughness Mode} \]
3. Simple Gravity Wave Drag Diagnostic

- Time Integrated Gravity Wave Drag per unit mass (GWD) gives the speed reduction of the mean flow:

\[
\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x} \left( \rho u^2 + p \right) + \frac{\partial (\rho uw)}{\partial z} = 0
\]

\[
\overline{(.)} = \frac{1}{L} \int_{0}^{L} (.) \, dx \quad \text{MF}_{x}
\]

\[
\frac{\partial \overline{u}}{\partial t} = -\frac{1}{\overline{\rho}} \frac{\partial (\overline{\rho u'w'})}{\partial z} = GWD
\]

\[
\Delta \overline{u}(z, t) = \int_{0}^{t} GWD(z, t') \, dt'
\]
Wave-Induced Wind Speed Reduction

Volume Mode Only

No Shear

Positive Shear

Negative Shear

Δ\(\bar{u}\)
\((m/s)\)

Final ΔU (m/s)

Time (minutes)

Height (km)
Wave-Induced Wind Speed Reduction

Volume Mode Only

No Shear

Positive Shear

Negative Shear

$\tau_b = \text{time to linear } z_{\text{break}}$

$\Delta \bar{u} (\text{m/s})$

$\tau_b \ 2\tau_b$

$\tau_b$

Final $\Delta U$ (m/s)
Wave-Induced Wind Speed Reduction

Volume Mode Only

No Shear
Momentum Deposition
Periodic in z
After Overturn

Positive Shear

Negative Shear

\[ \tau_b = \text{time to linear } z_{\text{break}} \]

\[ \Delta U (\text{m/s}) \]

\[ 2\tau_b \]

\[ \tau_b \]
Spectral Evolution in Negative Shear

Volume Mode
\( \lambda_x = 200 \text{ km} \)

Volume + Roughness Mode
\( \lambda_x = 28.5, 200 \text{ km} \)
Spectral Evolution in Negative Shear

**Volume Mode**
\[ \lambda_x = 200 \text{ km} \]

**Volume + Roughness Mode**
\[ \lambda_x = 28.5, 200 \text{ km} \]

- Roughness mode propagates up very quickly
- Smallest scales most important at high altitudes initially
Spectral Evolution in Negative Shear

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- Roughness mode breaks in negative shear
- Still propagates through despite overturning in negative shear
Spectral Evolution in Negative Shear

Volume Mode
$\lambda_x = 200$ km

$\lambda x \times = 200 \text{ km}$

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$\lambda_x = 28.5, 200$ km

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$\text{Spectral Evolution in Negative Shear}$
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Spectral Evolution in Negative Shear

Volume Mode
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Volume + Roughness Mode
$\lambda_x = 28.5, 200$ km

- Roughness mode breaks in negative shear
- Still propagates through despite overturning in negative shear
- Wavelength unchanged!
**Spectral Evolution in Negative Shear**

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\[ \lambda_x = 200 \text{ km} \]

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\[ \lambda_x = 28.5, 200 \text{ km} \]

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**Graphs**
- **u(x,z)**:
  - Zonal Wind (m s\(^{-1}\))
  - Height (km)
  - Distance (km)

- **MF\(_x(\lambda_x,z)\)**:
  - dMF/dln(Wavelength) (mPa)
  - Wavelength (km)
Spectral Evolution in Negative Shear

Volume Mode
\[ \lambda_x = 200 \text{ km} \]

Volume + Roughness Mode
\[ \lambda_x = 28.5, 200 \text{ km} \]

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Spectral Evolution in Negative Shear
Spectral Evolution in Negative Shear

Volume Mode
\( \lambda_x = 200 \text{ km} \)

Volume + Roughness Mode
\( \lambda_x = 28.5, 200 \text{ km} \)

- Volume mode reaches negative shear
Spectral Evolution in Negative Shear

Volume Mode
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Volume + Roughness Mode
\( \lambda_x = 28.5, 200 \text{ km} \)

- Volume mode reaches negative shear
- Large scale -\( u' \) by volume mode causes further attenuation of roughness mode!
Spectral Evolution in Negative Shear

Volume Mode
\( \lambda_x = 200 \text{ km} \)

Volume + Roughness Mode
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**Volume + Roughness Mode**

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Spectral Evolution in Negative Shear

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Volume Mode
\[ \lambda_x = 200 \text{ km} \]

Volume + Roughness Mode
\[ \lambda_x = 28.5, 200 \text{ km} \]

- Volume mode reaches negative shear
- Large scale \(-u'\) by volume mode causes further attenuation of roughness mode!
Wave-Induced Wind Speed Reduction

No Shear

Positive Shear

Negative Shear

$\tau_b = \text{time to linear } z_{\text{break}}$

$\tau_b$, $2\tau_b$

Volume+Roughness

Volume Only

$\Delta \bar{u}$ (m/s)

Final $\Delta U$ (m/s)

$\tau_b = \text{time to linear } z_{\text{break}}$
Influences of Vertical Shear

• Attenuation/Momentum Deposition periodic in z

• Negative Shear - attenuation confined to region of negative shear

• No Shear - attenuation periodic in z

• Positive Shear - more spaced out regions of overturning aloft
Scale Interaction

• Roughness Mode ($\lambda_x=28.5\text{km}$)
  – Carries more momentum flux than the Volume Mode ($\lambda_x=200\text{km}$)
  – Propagates up $\approx 7$ times faster than Volume Mode
    • Smaller scales more important initially aloft in MW events
  – Attenuation initially smoothly distributed in vertical

• Eventually, Volume Mode controls attenuation for both modes
  – Larger scale $u'$ from Volume Mode influences background for smaller scale Roughness Mode
  – Attenuation periodic in vertical for both modes
Idealized Wind Profiles

\[ U(z) = U_{z_0} \left( z + Z_1 e^{-\left( \frac{z}{Z_1} \right)} - Z_1 \right) - \frac{\Delta U}{2} \left( 1 + \tanh \left( \frac{\pi \left( z - z_i \right)}{Z_2} \right) \right) + U_0 \]

- **Term 1** – Positive Shear Term
  - \( U_{z_0} = 1 \text{ m s}^{-1} \text{ km}^{-1} \) – Upper Shear

- **Term 2** – Negative Shear Term
  - \( \Delta U \) – total wind speed reduction

- **Term 3** – Surface Wind: \( U_0 \)

---

No Shear: \( U_{z_0} = 0, \quad \Delta U = 0 \text{ m/s} \quad U_0 = 30 \text{ m/s} \)

Positive Shear: \( U_{z_0} = 1 \text{ m/s/km} \quad \Delta U = 0 \quad U_0 = 30 \text{ m/s} \)

Negative Shear: \( U_{z_0} = 1 \text{ m/s/km} \quad \Delta U = 30 \text{ m/s} \quad U_0 = 30 \text{ m/s} \)
Volume Mode $\Delta U$

**No Shear**

**Positive Shear**

**Negative Shear**

$\tau_b = \text{time to linear } z_{\text{break}}$

$\Delta \bar{u} \quad \text{(m/s)}$

$\tau_b \quad 2\tau_b$

$\tau_b \quad 2\tau_b$
\( \tau_b = \text{time to linear } z_{\text{break}} \)
\( \tau_b = \text{time to linear } z_{\text{break}} \)
Volume + Roughness Mode GWD

No Shear

Positive Shear

Negative Shear

$\tau_b$ = time to linear $z_{break}$

$2\tau_b$

$\tau_b$

Time (minutes)

Time Avg (m/s/day)

GWD (m/s/day)
Linear Theory Predicts $z_b$?

- Waves overturn where $u'$ most negative
- Most negative $u'$ at $x = 0$, $\varphi = \frac{3}{4}2\pi + 2\pi n$

WKB phase at $x = 0$:

$$\phi(z) = \int_{0}^{z} \frac{N(z')}{U(z')} \, dz'$$
Linear Theory Predicts $z_b$?

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WKB phase at $x = 0$:

$$\phi(z) = \int_0^z \frac{N(z')}{U(z')} \, d\tilde{z}'$$
Momentum Deposition by Mode

Volume Mode Only
\[ \lambda_x = 200 \text{ km} \]
\[ t = 2\tau_b = 7-10 \text{ hr} \]

Volume + Roughness Mode
\[ \lambda_x = 28.5 \text{ km} \]
\[ t = 2\tau_b = 1-1.33 \text{ hr} \]
Momentum Deposition by Mode

@ Volume Mode Overturning Time:
\[ t = 2\tau_b = 7-10 \text{ hr} \]

No Shear

Positive Shear

Negative Shear

Solid: Volume Mode
Dotted: Volume + Roughness Mode
Background/Motivation

$z \approx 28 \text{ km}$

WRF Var(T)

Observed AIRS Var(T)

Deep propagation observed as well!

Kruse and Smith 2016
Mountain Wave Overturning

- \( t = 2\tau_b + 1 \text{ hr} \)
- \( 2\tau_b \approx 7-10 \text{ hr}, 1 \text{ hr} \approx 12 \text{ buoyancy periods}, t \approx 8-11 \text{ hr} \)

No Shear: Many overturning levels
Positive Shear: Fewer, spaced overturning levels
Negative Shear: Overturning confined to negative shear layer

Color Shading: Zonal Wind
Contours: Isentropes
Linear Theory Predicts $z_b$?

- Waves overturn where $u'$ most negative
- Most negative $u'$ at $x = 0$, $\varphi = \frac{3}{4}2\pi + 2\pi n$

WKB Phase @ $x=0$: $\phi(z) = \int_{0}^{\hat{z}} \frac{N(z')}{U(z')} \, dz'$

Number of Vertical Wavelengths:

$$\hat{z} = \frac{\phi(z)}{2\pi} = \frac{1}{2\pi} \int_{0}^{\hat{z}} \frac{N(z')}{U(z')} \, dz'$$

Should break @ $\hat{z} = \frac{3}{4} + n$