# Reversible and Irreversible Mountain Wave Momentum Deposition in Sheared Environments

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**Primary Research Questions:**

1. How do initially linear mountain waves (MWs) propagate, breakdown, and influence their environment in a MW event?
   - Influences of vertical shear? Scale?

2. How important is reversible GWD by MWs?

3. In what ways might GWD parameterizations be improved?

# MF, GWD, and ΔU

- Time Integrated Gravity Wave Drag per unit mass (GWD) gives the mean flow reduction:

$$\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x}\left(\rho u^2 + p\right) + \frac{\partial(\rho u w)}{\partial z} = 0$$

$$\overline{(\,.\,)} = \frac{1}{L}\int_0^L (\,.\,)\, dx \qquad \overline{u} = U$$

Considering models **periodic in x!**

$$MF = \overline{\rho}\,\overline{u'w'}$$

$$\frac{\partial U}{\partial t} = -\frac{1}{\overline{\rho}}\frac{\partial MF}{\partial z} = GWD$$

$$\Delta U(z,t) = \int_0^t -\frac{1}{\overline{\rho}}\frac{\partial MF}{\partial z}\, dt'$$

# Reversible and Irreversible ΔU

- In MW events, MWs interact with their environment both reversibly and irreversibly
- **Reversible (Non-Dissipative) $\Delta U = \Delta U_{rev}$:**
  - Mean flow reduction that occurs as MWs propagate into a previously undisturbed flow
  - If MW forcing finite in time and MWs do not dissipate, MWs return ambient flow back as they propagate out of the layer; hence, this interaction is reversible
- **Irreversible (Dissipative) $\Delta U = \Delta U_{irr}$:**
  - Mean flow reduction that occurs as MWs dissipate/break, which irreversibly alters the mean flow

$$\Delta U = \Delta U_{rev} + \Delta U_{irr}$$

# Tools

1. Non-Linear Model: **WRF**
   - Resolves waves and their non-linear breakdown
   - **Periodic domain** allows diagnosis of **total $\Delta U = \Delta U_{rev} + \Delta U_{irr}$**
2. Linear Model: **Fourier Ray** (Broutman et al. 2002)
   - Spectral, quasi-transient, non-coupled/steady background
   - Allows diagnosis of **reversible $\Delta U = \Delta U_{rev}$**
3. Saturation Parameterization: **Lindzen Type** (Lindzen 1981, McFarlane 1987)
   - Monochromatic, instantaneous propagation, waves not allowed to reach overturning amplitude (wave saturation)
   - Gives estimate of **irreversible GWD, $\Delta U_{irr}$** in most coarse models

**WRF** **Fourier Ray** **Param**

$$\Delta U = \Delta U_{rev} + \Delta U_{irr}$$

# Linear Fourier Ray Model

- Compute ray solution in Fourier space, then invert

$$\hat{\eta}(k,z) = \left(\frac{\overline{\rho}_0}{\overline{\rho}(z)}\right)^{1/2} \hat{h}(k) \left[\frac{c_{gz_0}(k)}{c_{gz}(k,z)} \frac{U(z)}{U_{0_m}}\right]^{1/2} e^{i\int_0^z m(k,z')dz'}$$

Eckermann et al. 2015

$$\eta(x,z,t) = \int_{-\infty}^{\infty} F_{sfc}(t - t_{prop}(k,z))\hat{\eta}(k,z)e^{ikx}\,dk$$

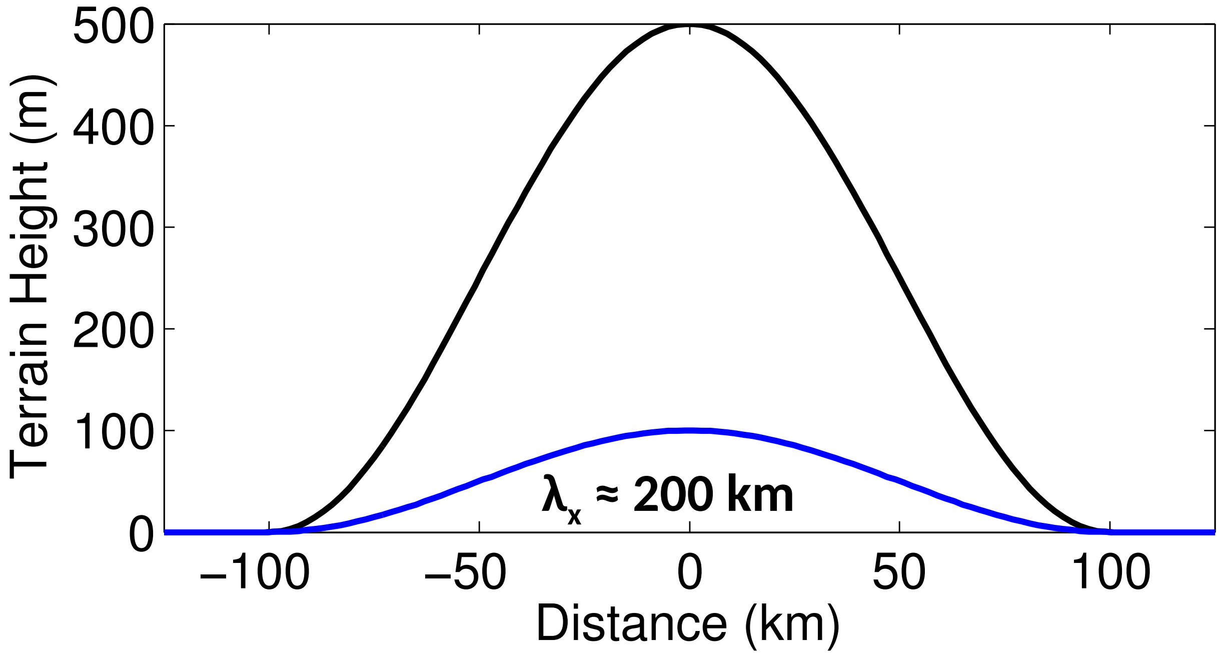
$$F_{sfc}(t) = \frac{U_0(t)}{U_{0_m}} \qquad t_{prop}(k,z) = \int_0^z \frac{dz'}{c_{gz}(k,z')}$$

- Terrain, h, provides scales and z = 0 amplitudes
- Wave action conservation and density modify these amplitudes in altitude
- **Quasi-Transient:**
  - **$c_p$ = 0** for all scales
  - Transience due to Surface Forcing, $F_{sfc}$, which takes into account $c_{gz}$ spectrum and arbitrary cross-barrier flow function, $U_0(t)$
- Evanescent, reflected waves neglected
- **Waves NOT coupled to ambient flow**

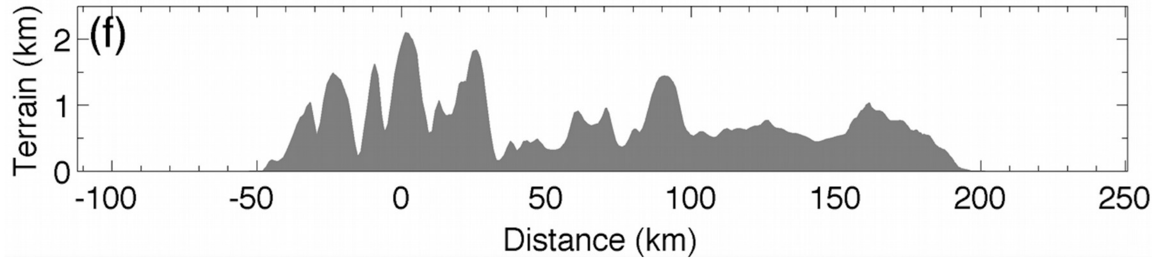
# Idealized Terrain

$$h(x) = \begin{cases} 0.5h_m(1+\cos(kx)) & , \quad |x| \le d \\ 0 & , \quad |x| > d \end{cases}$$

- $k = \pi/d$, $d = 100$ km
- $h_m$: max terrain height
  - **$h_m$ = 50 m, 500 m**
- **Compact Terrain: results in a broad(-ish) spectrum**




**New Zealand Transect**

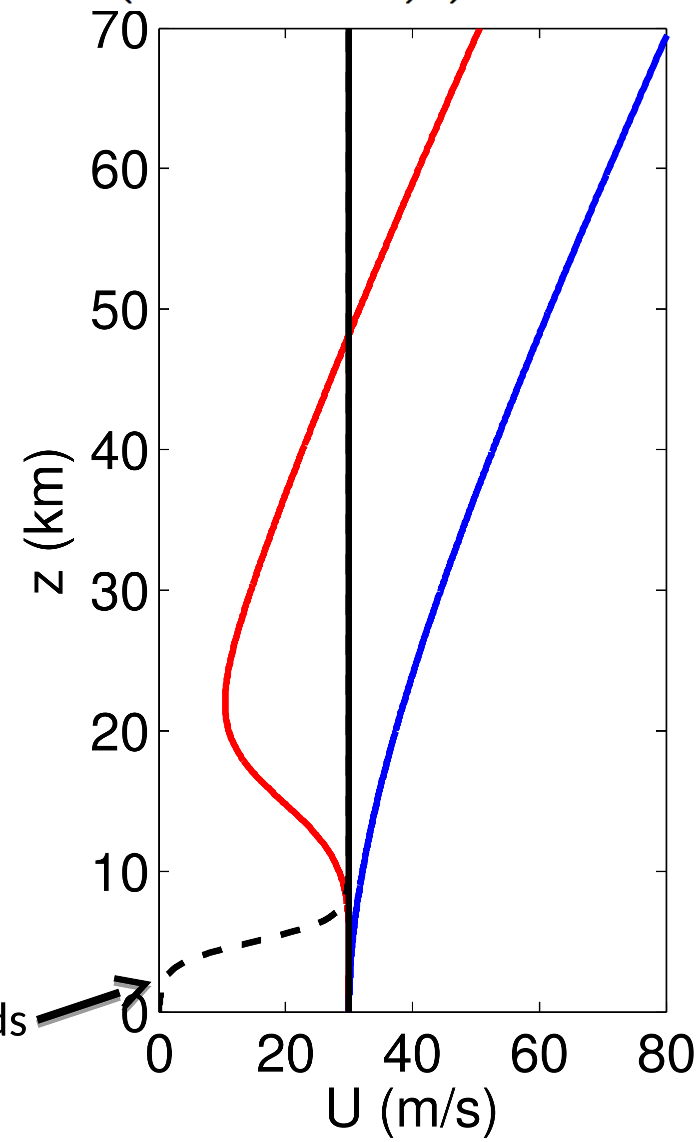


# Wind Profiles

$$U(z) = U_{z_0}\left(z + Z_1 e^{-\frac{z}{Z_1}} - Z_1\right) - \frac{\delta U}{2}\left(1 + \tanh\left(\frac{\pi}{2}\frac{(z - Z_i)}{Z_2}\right)\right) + U_0$$

- Upper Shear: **$U_{z0}$**
- Ambient Wind Reduction: **$\delta U$**
- Surface Wind: **$U_0$ = 30 m/s**

| | | |
|---|---|---|
| **No Shear:** | **$U_{zo}$ = 0,** | **$\delta U$ = 0 m/s** |
| **Positive Shear:** | **$U_{zo}$ = 1 m/s/km** | **$\delta U$ = 0 m/s** |
| **Negative Shear:** | **$U_{zo}$ = 1 m/s/km** | **$\delta U$ = 30 m/s** |



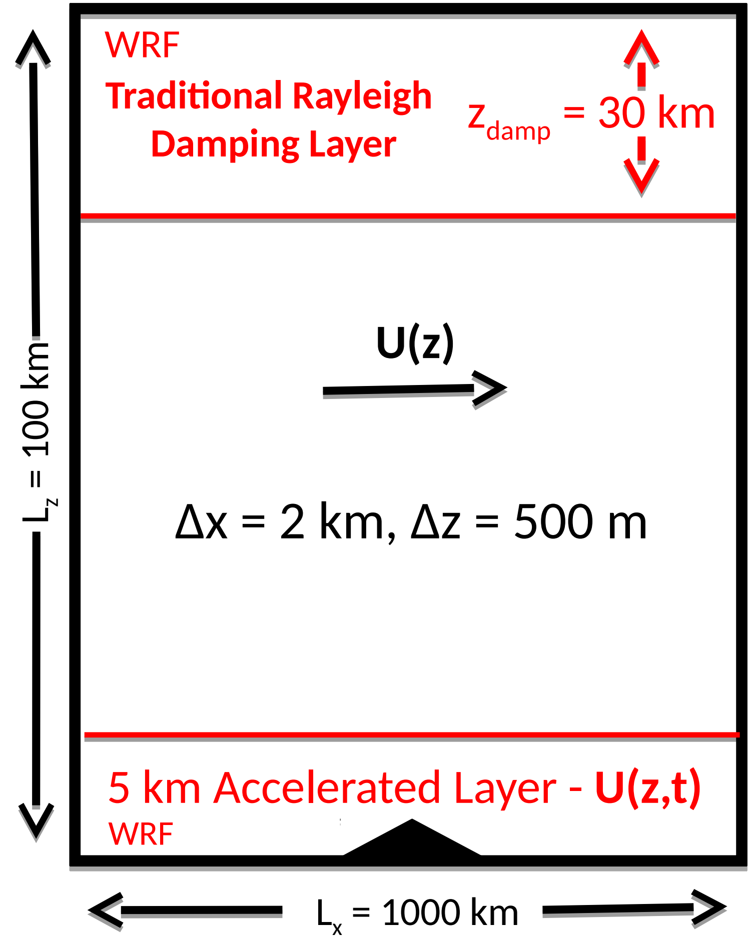

Initial low-level winds

# Domains, Event Forcing

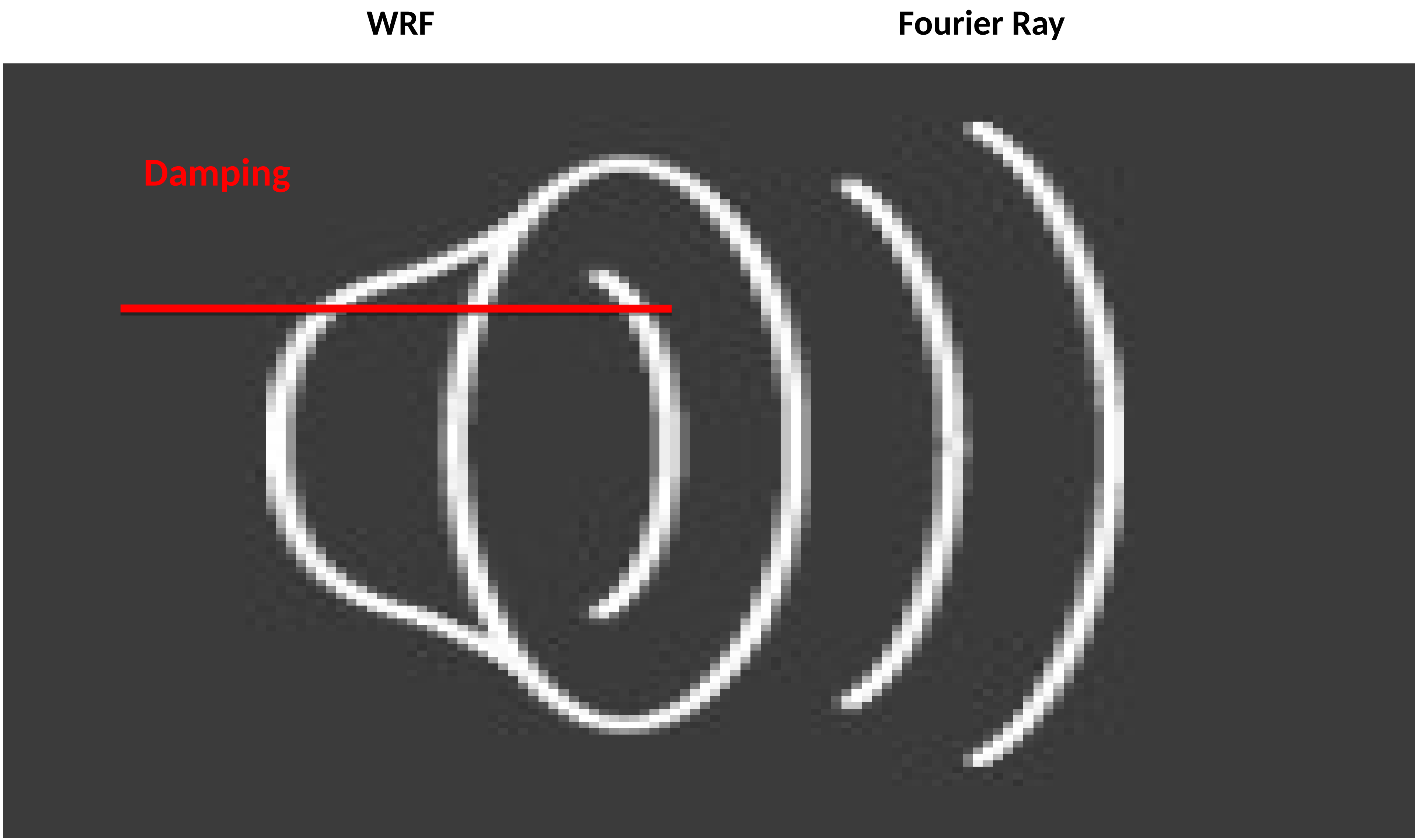
- **Setup**
  - 2-D
  - **Horizontally Periodic**
  - Constant N = 0.02 $s^{-1}$
  - f = 0
  - Inviscid

- **MW Event Forcing (12 hr)**
  - **WRF:** Wind in lowest 5 km uniformly accelerated from zero to desired profile in 20 minutes, allowed to evolve for 12 hours, then decelerated back to zero
  - **FR, GWD Parameterization:** Same surface-level winds as WRF

**Model Domains**



# Linear Mountain Wave Evolution




No Shear, $h_m$ = 50 m

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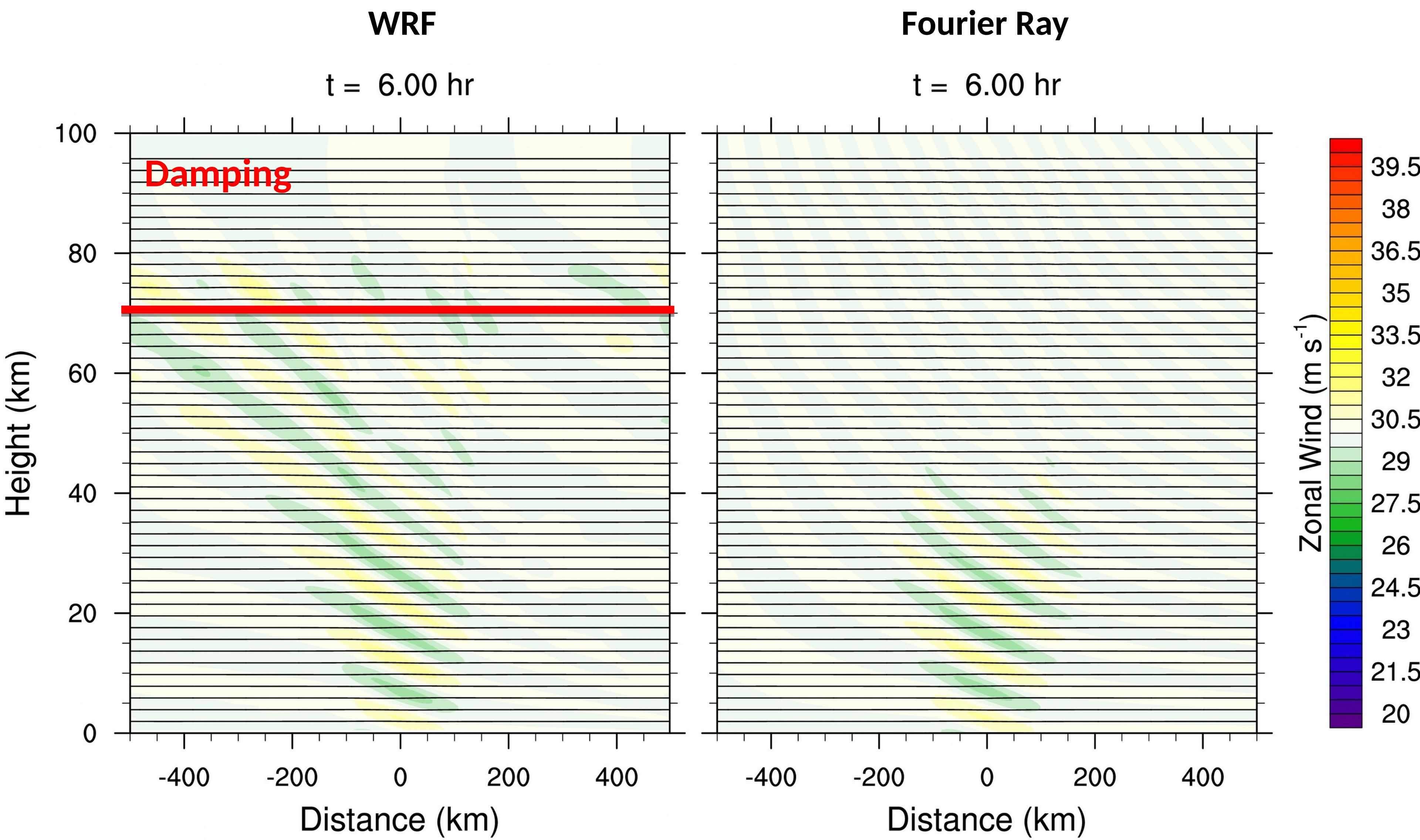


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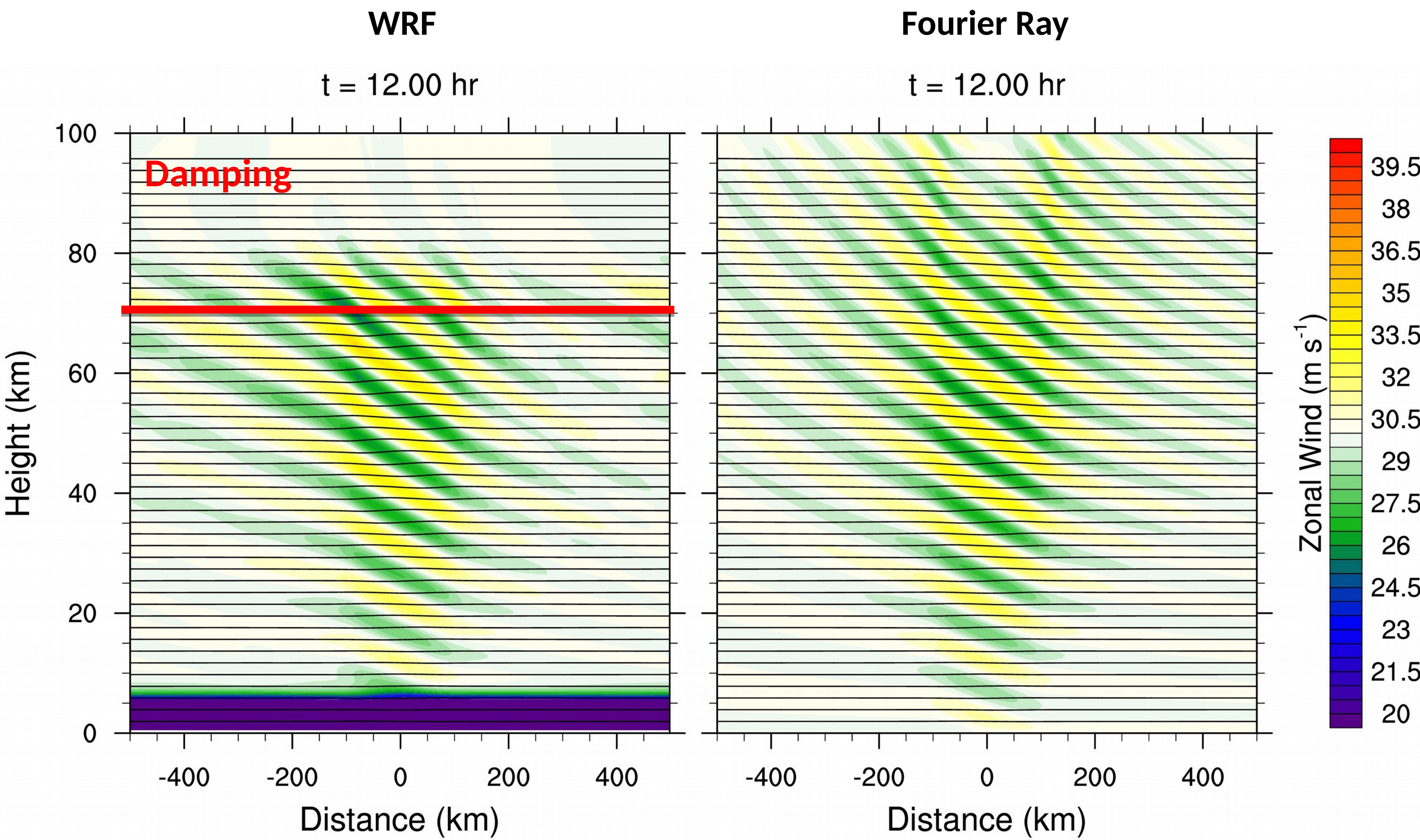


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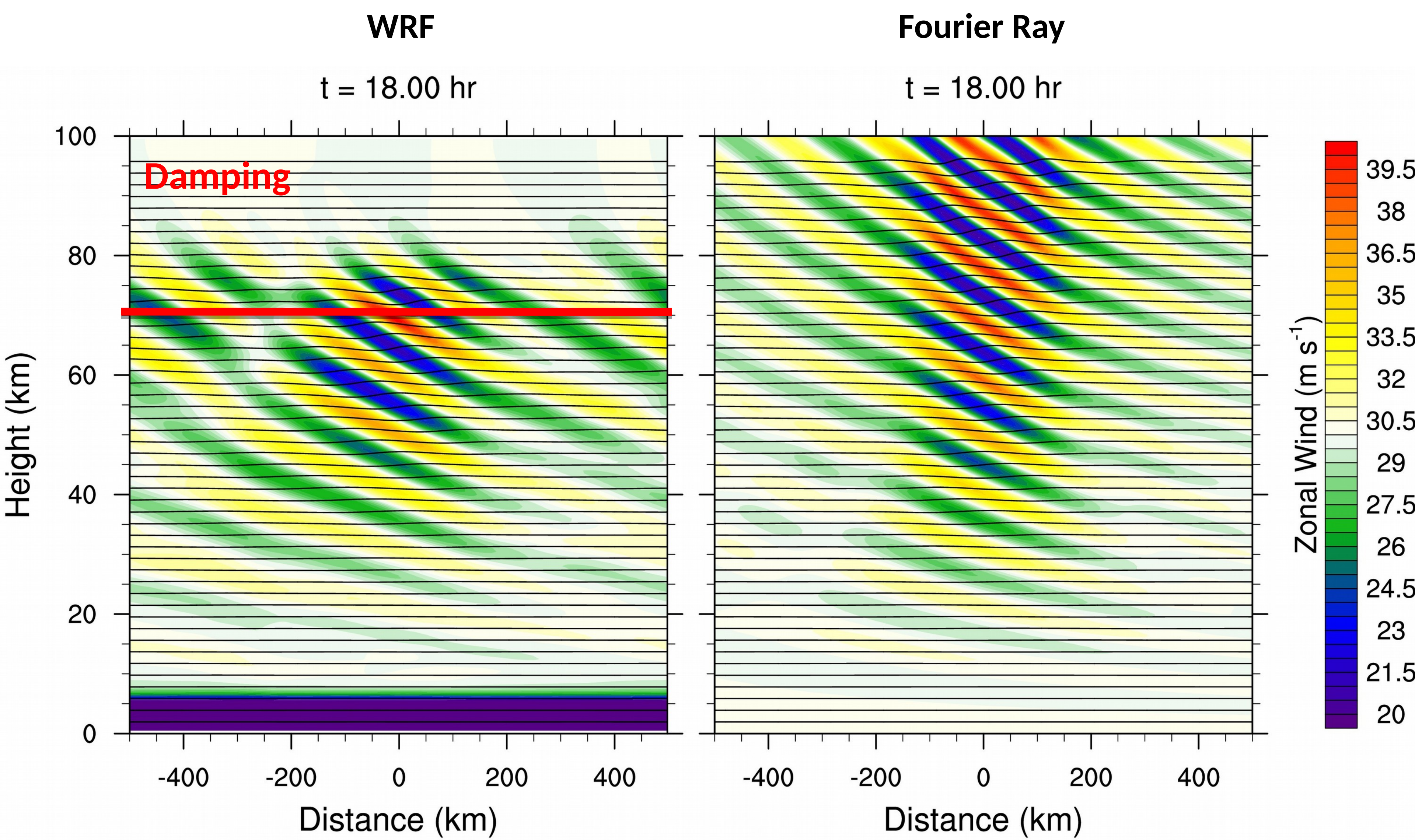


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# Wave Scales, Initial MF Evolution

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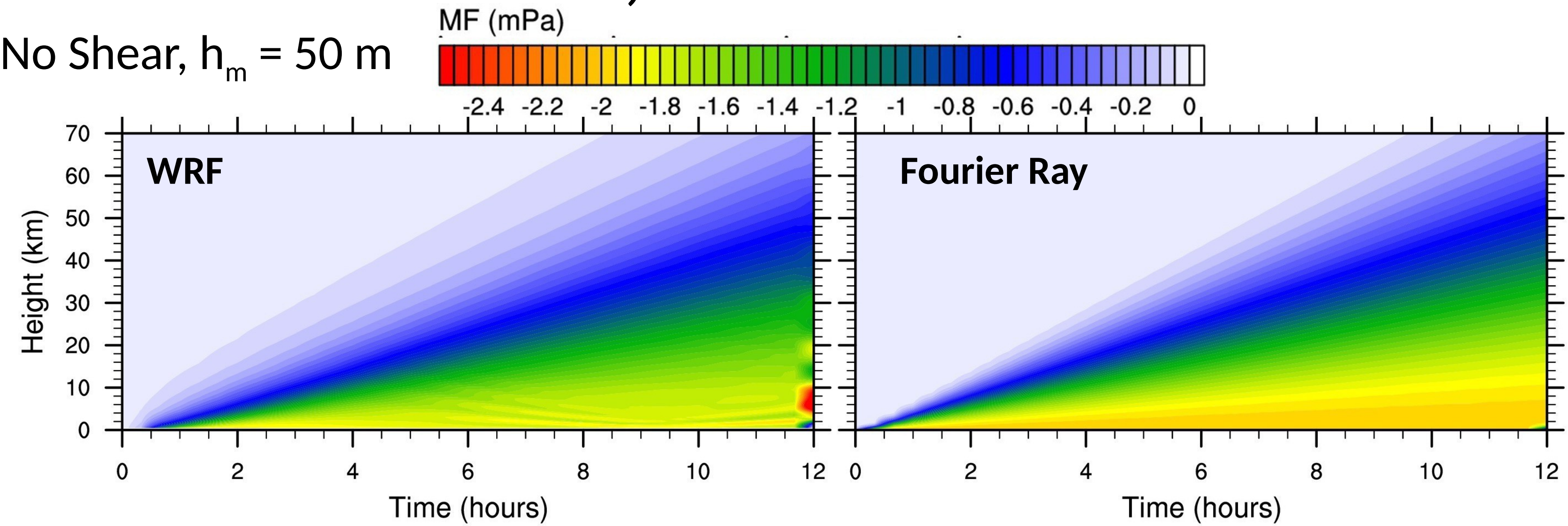


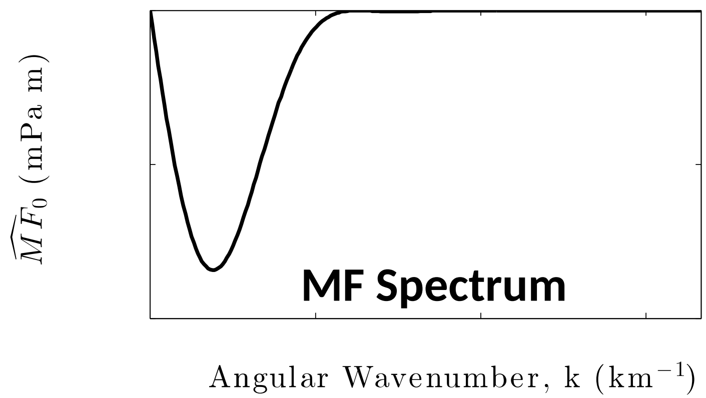


$$\widehat{MF}_0(k) = -\bar{\rho} N U |k| |\hat{h}(k)|^2$$

$$c_{gz} = \frac{U^2|k|}{N}$$

- MW generation produces non-dissipative MF gradient initially
- Spectrum and $c_{gz}$ dispersion spread MF profiles vertically in time
  - **Long waves propagate up slowly, short waves quickly**

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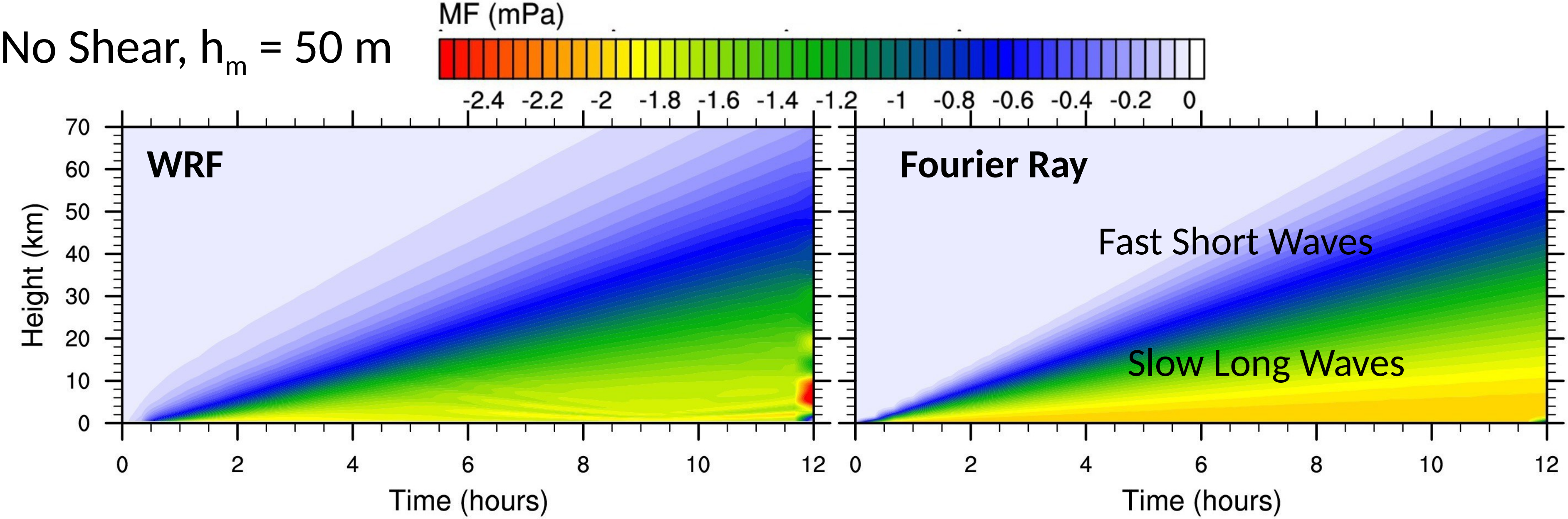


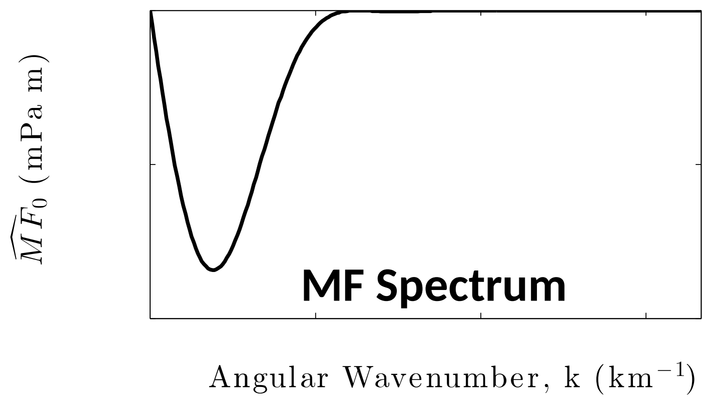


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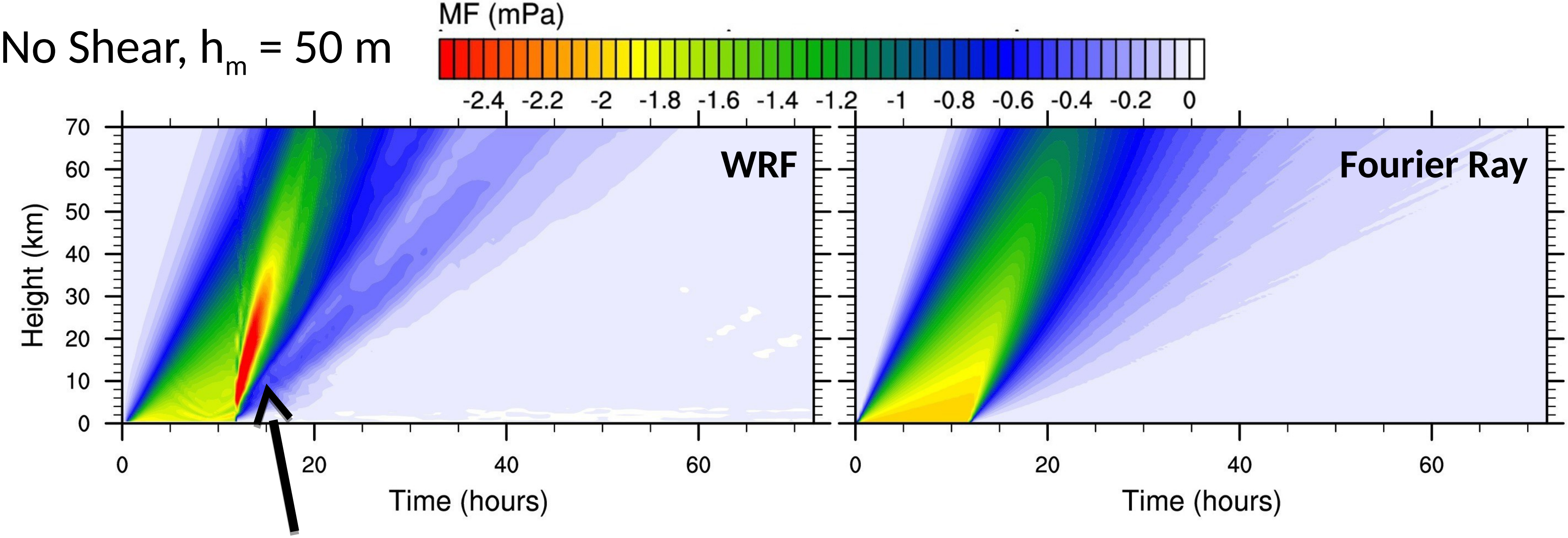
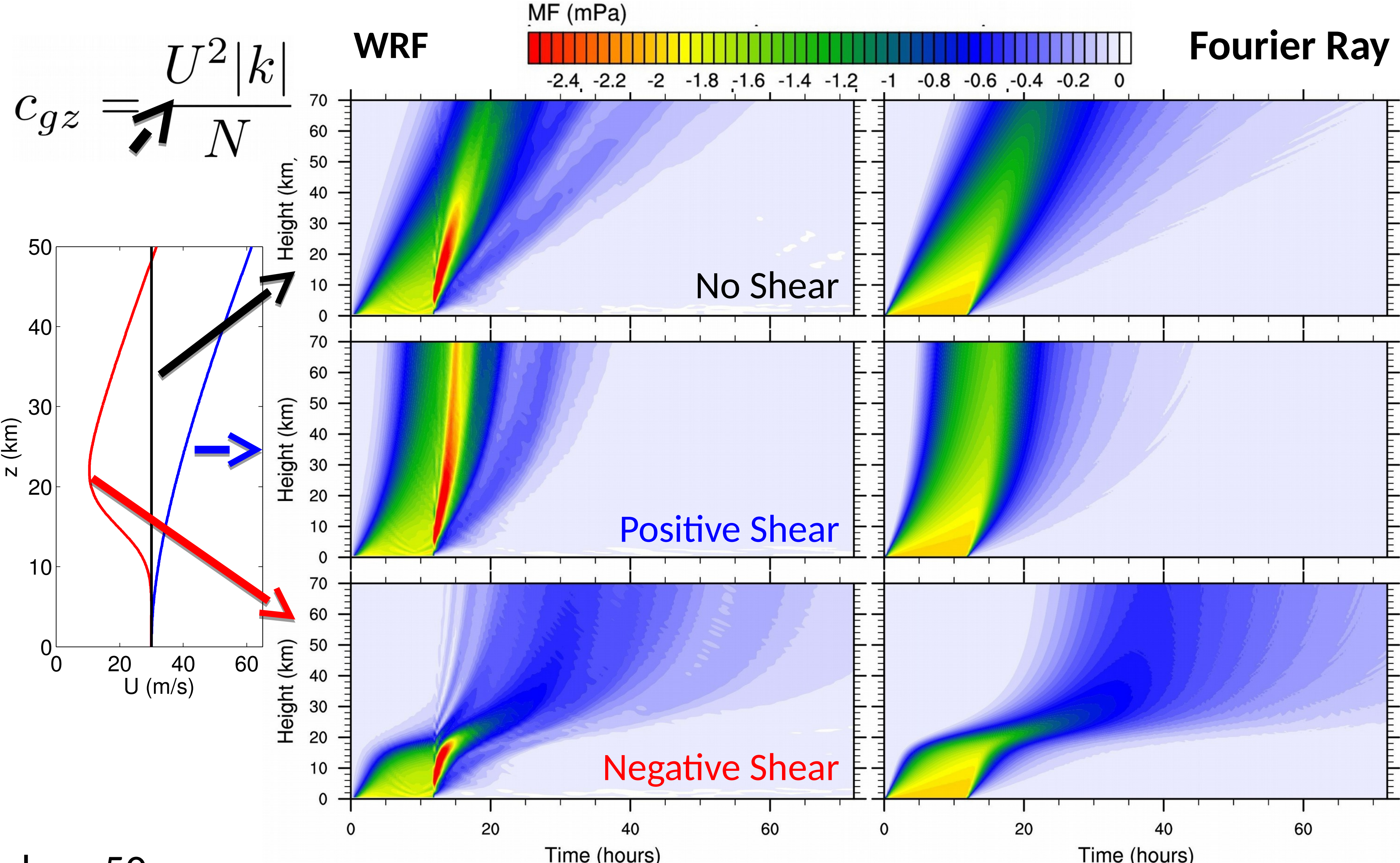



- MF maximum associated low-level deceleration at end of event in WRF
  - Low-level wave field suddenly travelling upstream ($c_p$≈-30m/s)
  - Termed “travelling wave MF maximum” here
  - Physics of this feature not fully understood yet
  - Not present in FR solutions because of $c_p$ = 0 constraint
- Other than the travelling wave feature, good quantitative agreement between WRF and FR

# Shear Effects on MF Evolution

- Positive (negative) shear spreads (compresses) $MF_x$ in vertical

$$c_{gz} = \frac{U^2|k|}{N}$$




$h_m$ = 50 m

# $\Delta U_{rev}$ in Fourier Ray Solutions

- $\Delta U_{rev}$ can be computed in two equivalent ways:

1. From the time integral of MF gradient:

$$\Delta U = \int_0^t -\frac{1}{\overline{\rho}}\frac{\partial MF}{\partial z}\, dt'$$

$$MF = \frac{1}{2\pi}\int_{-\infty}^{\infty} \widehat{MF}(k)\, dk$$

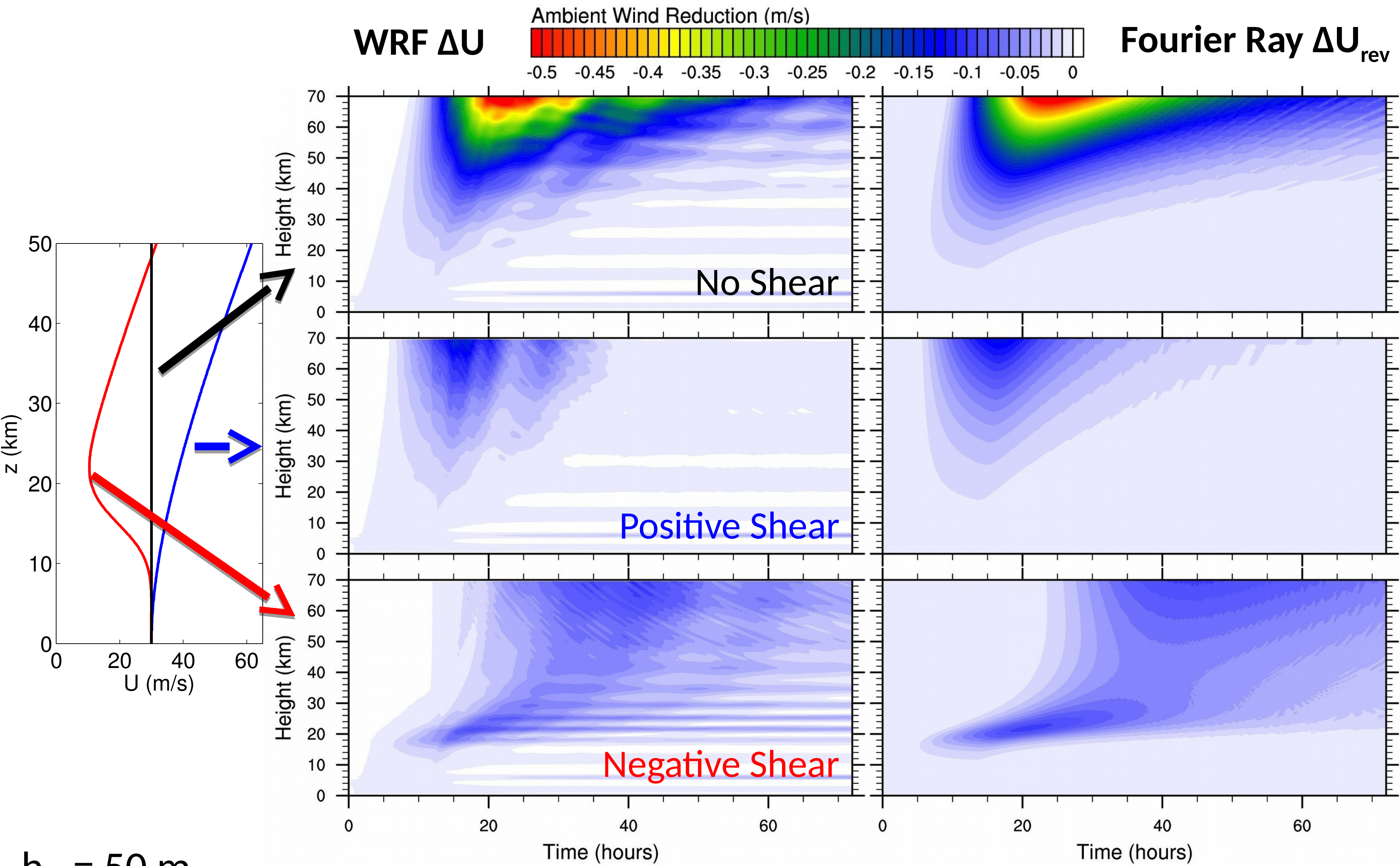
2. Or, simply from the MF present:

$$\Delta U_{rev}(z,t) = \frac{1}{2\pi\overline{\rho}}\int_{-\infty}^{\infty} \frac{\widehat{MF}}{c_{gz}}\, dk$$

- Follows from Parseval's theorem + linear theory, or alternatively Stokes' Theorem (Sutherland 2010)
- Used the 2nd spectral method to compute $\Delta U_{rev}$ in the Fourier Ray solutions

# $\Delta U_{rev}$ Evolution

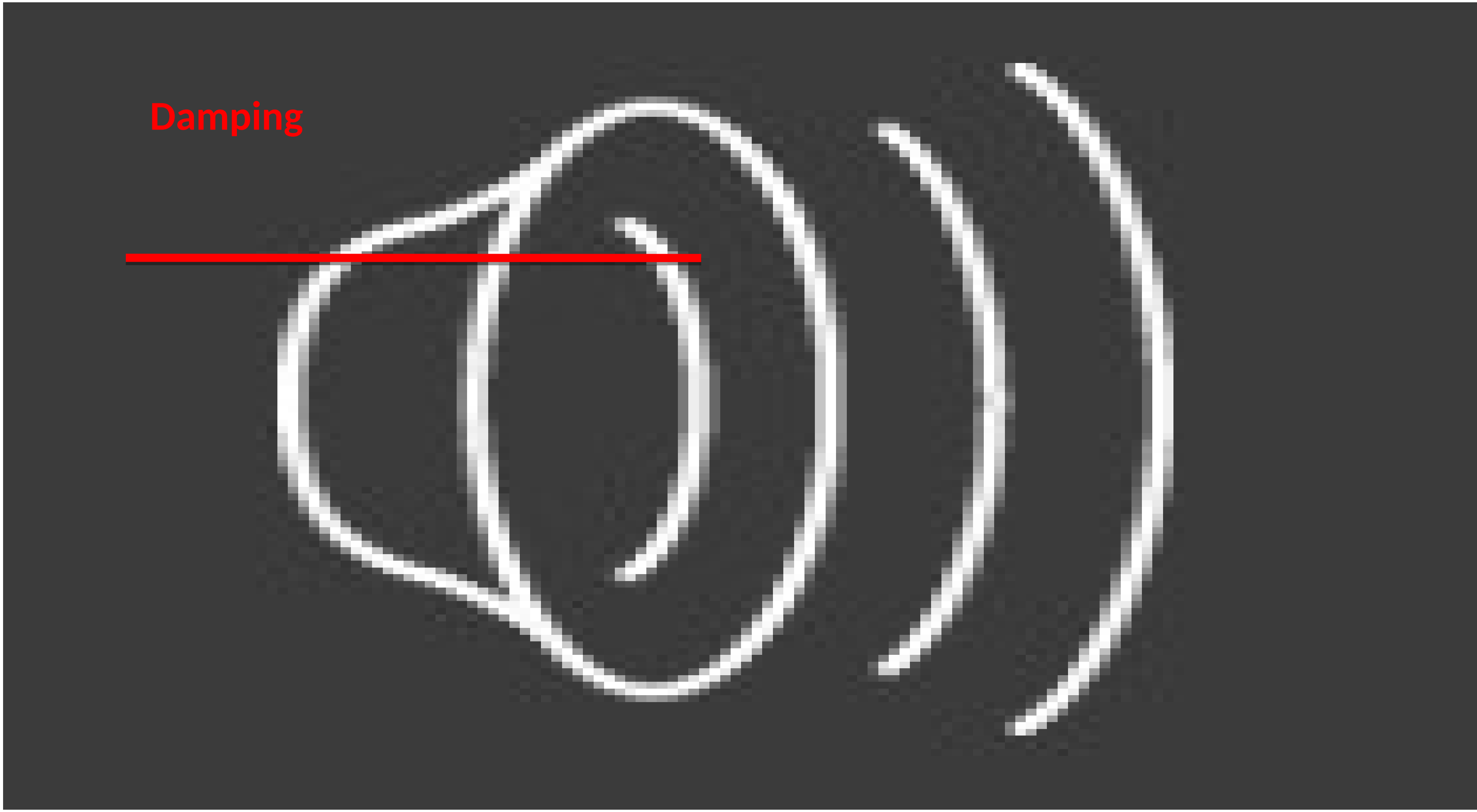
- Is non-dissipative ΔU reversible? **Yes**, but can take several days




$h_m$ = 50 m

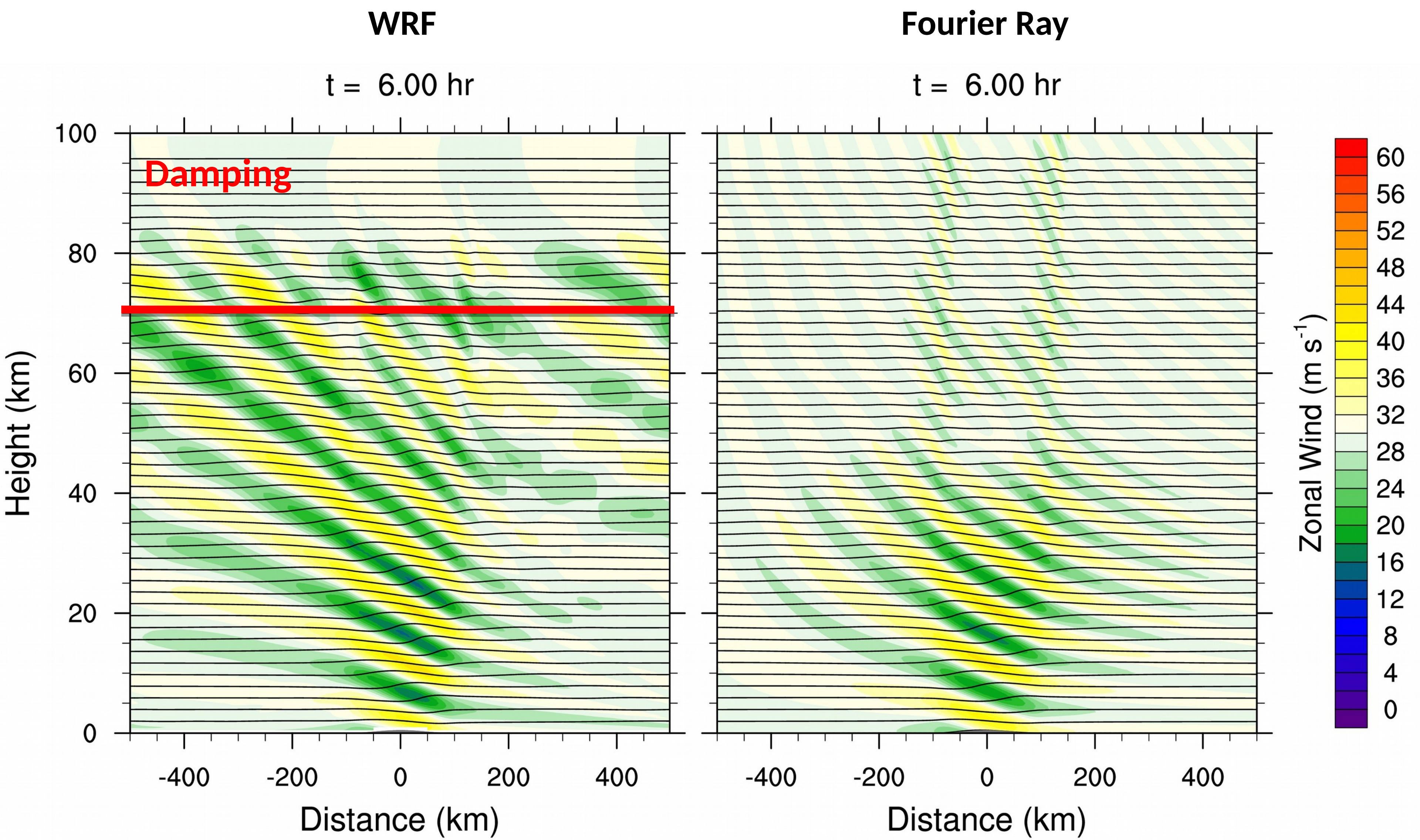
# Breaking MW Evolution

**WRF** | **Fourier Ray**




$h_m$ = 500 m

# Breaking MW Evolution

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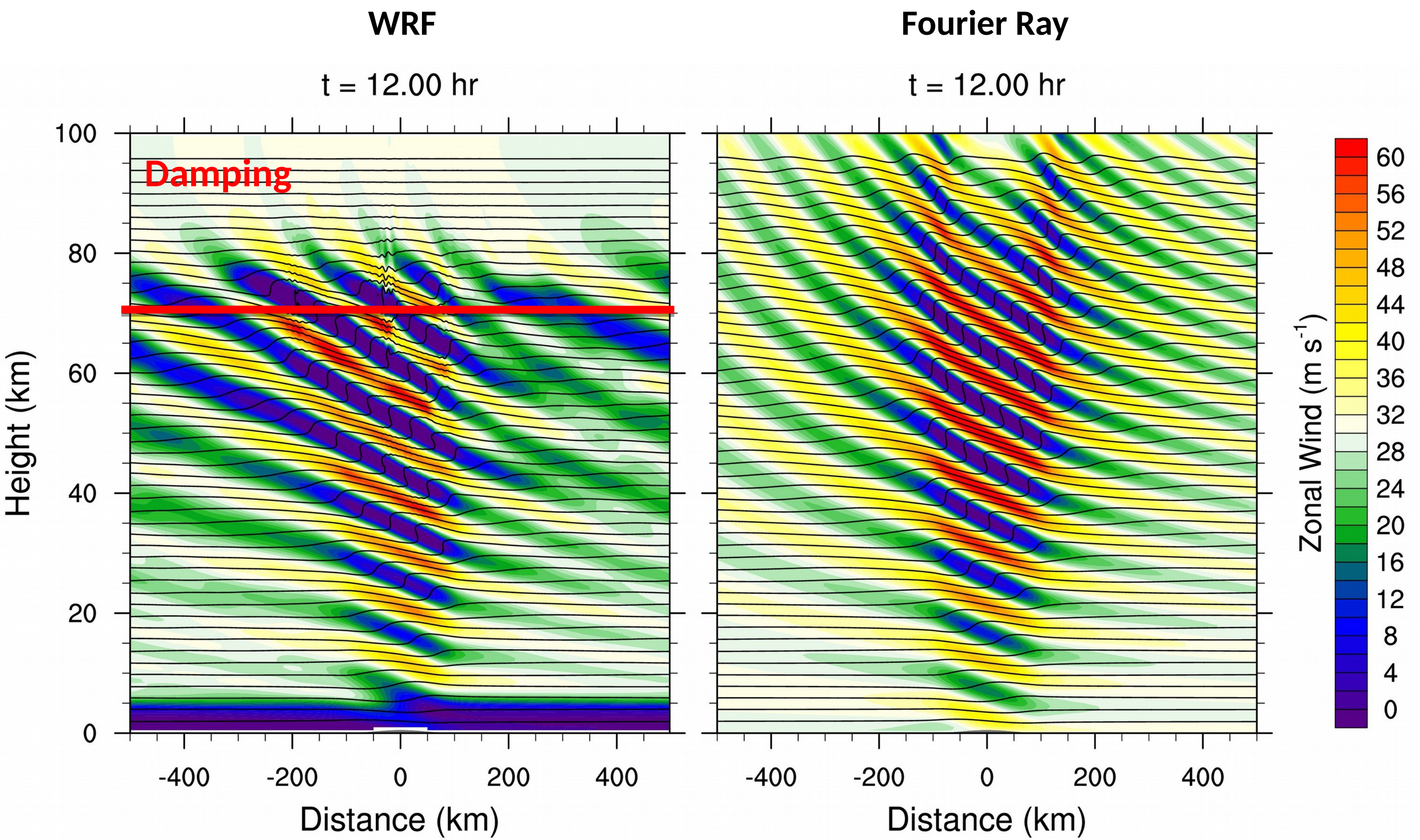


$h_m$ = 500 m

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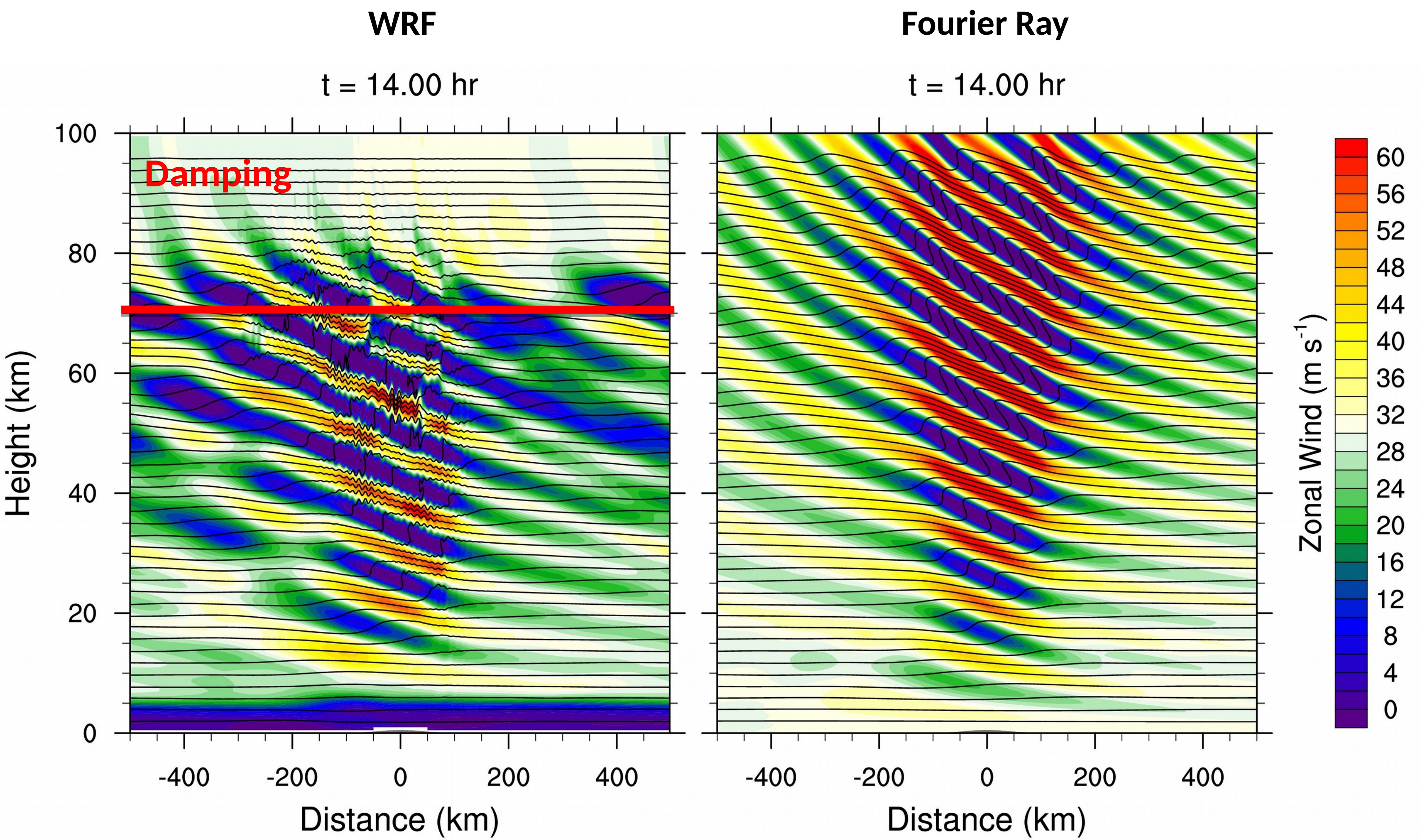
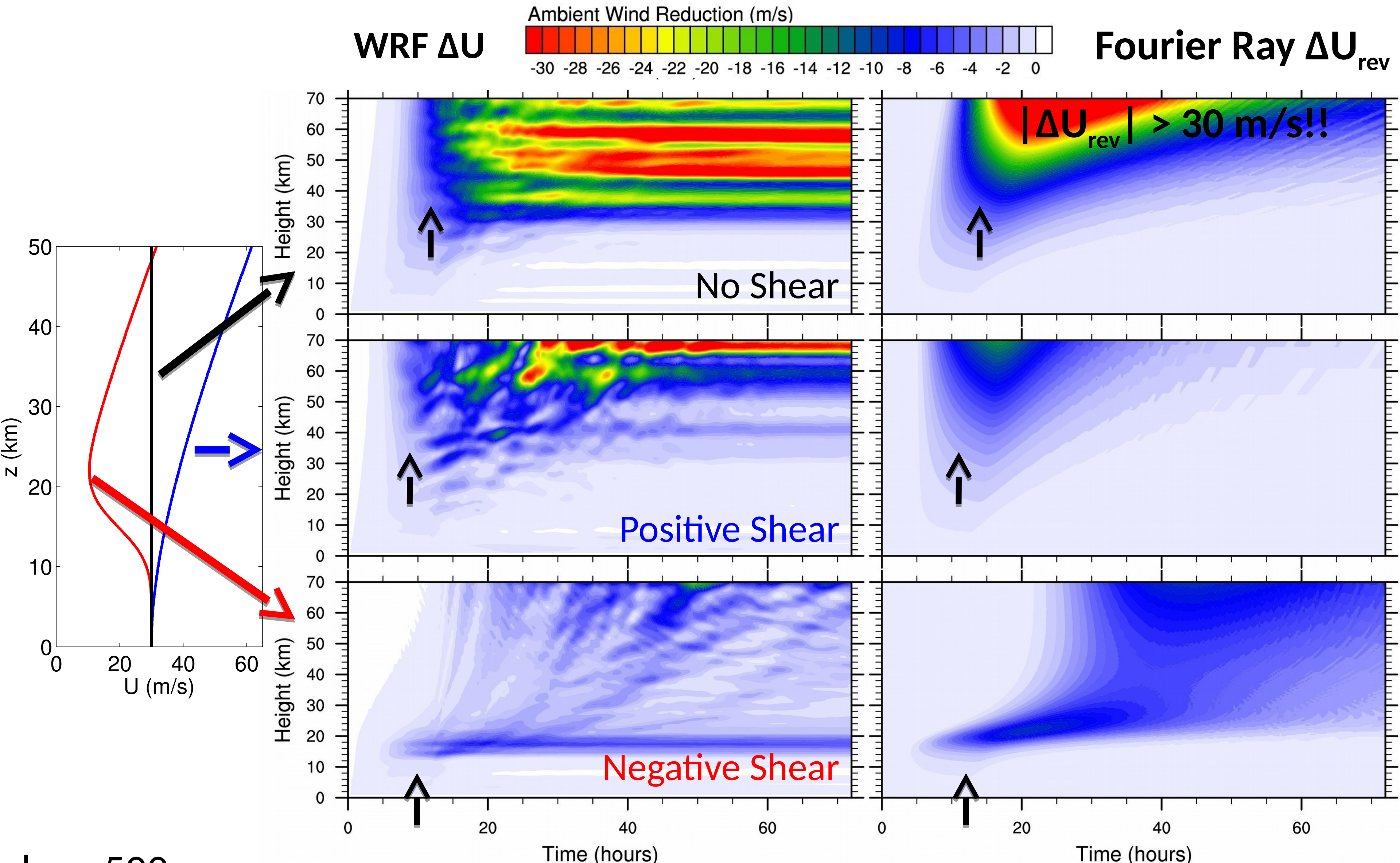



$h_m$ = 500 m

# ΔU Evolution with Breaking

- Substantial $\Delta U_{rev}$ (5-10 m/s) prior to breaking




$h_m$ = 500 m

# MW Drag Parameterization

- Assumptions: **Monochromatic** ($\lambda_x$=200km), **instant propagation**, **wave amplitude saturates**, steady ambient ($\Delta U_{rev}$=0), vertical propagation only, 2-D, hydrostatic, no lateral variations…

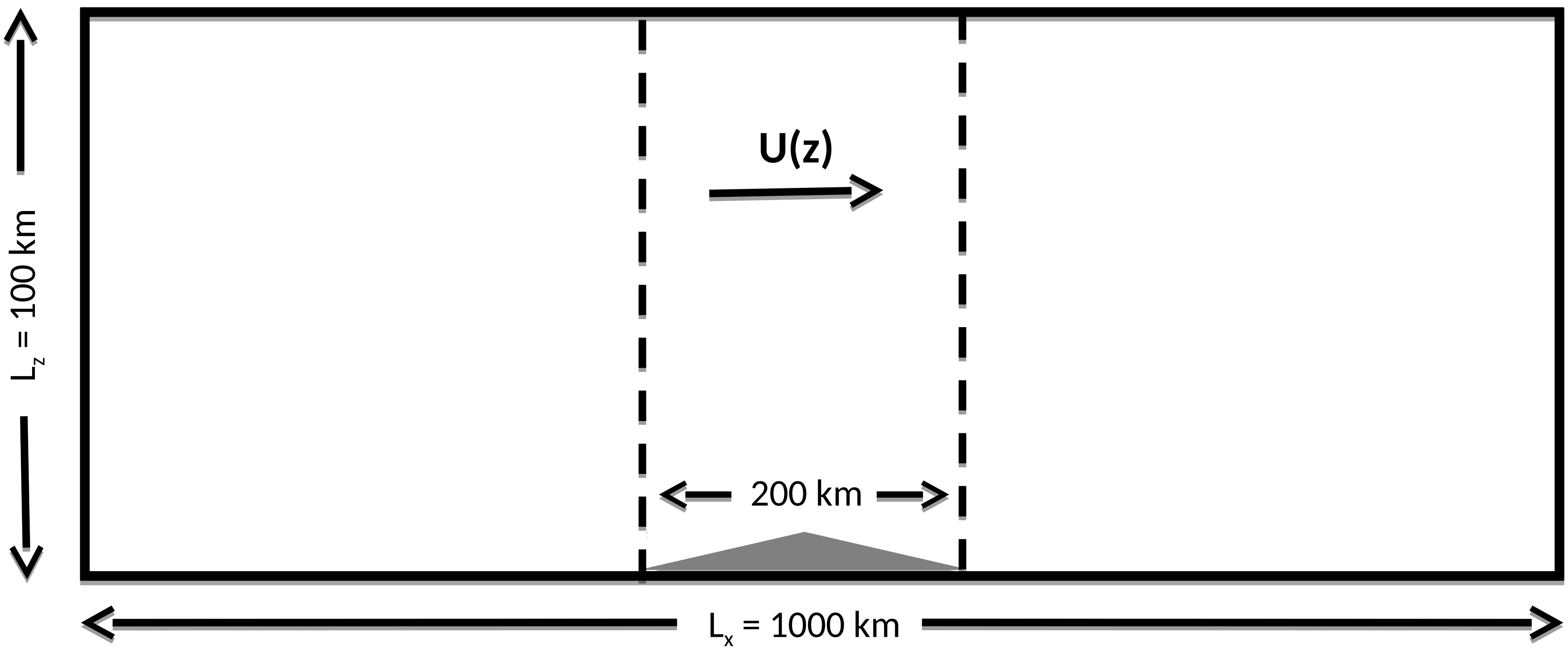
1. Determine MF (next slide), u' amplitude at surface
2. Determine u' amplitude, MF at next model level
   - Compute u' amplitude above via MF conservation
     - If u'<= U(z): no dissipation, ΔMF/Δz = 0
     - If u'> U(z): set u'=U(z), compute new MF
3. Iterate up through all model levels
4. **10-km Vertical Moving Avg Smoother Applied to MF(z)**
   - Necessary! Enforces vertical scale of dissipation.
5. ΔMF/Δz, ρ(z) used to compute GWD

# MW Parameterization Domain

- Average surface MF computed from full terrain spectrum:

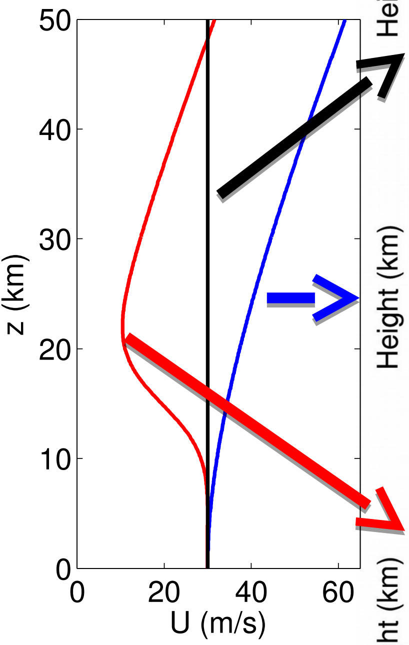
$$MF_0 = -\frac{\overline{\rho}_0 N U_0}{4\pi L} \int_{-\infty}^{\infty} \left(1 - \frac{U^2 k^2}{N^2}\right)^{1/2} |k||\hat{h}|^2 \, dk$$

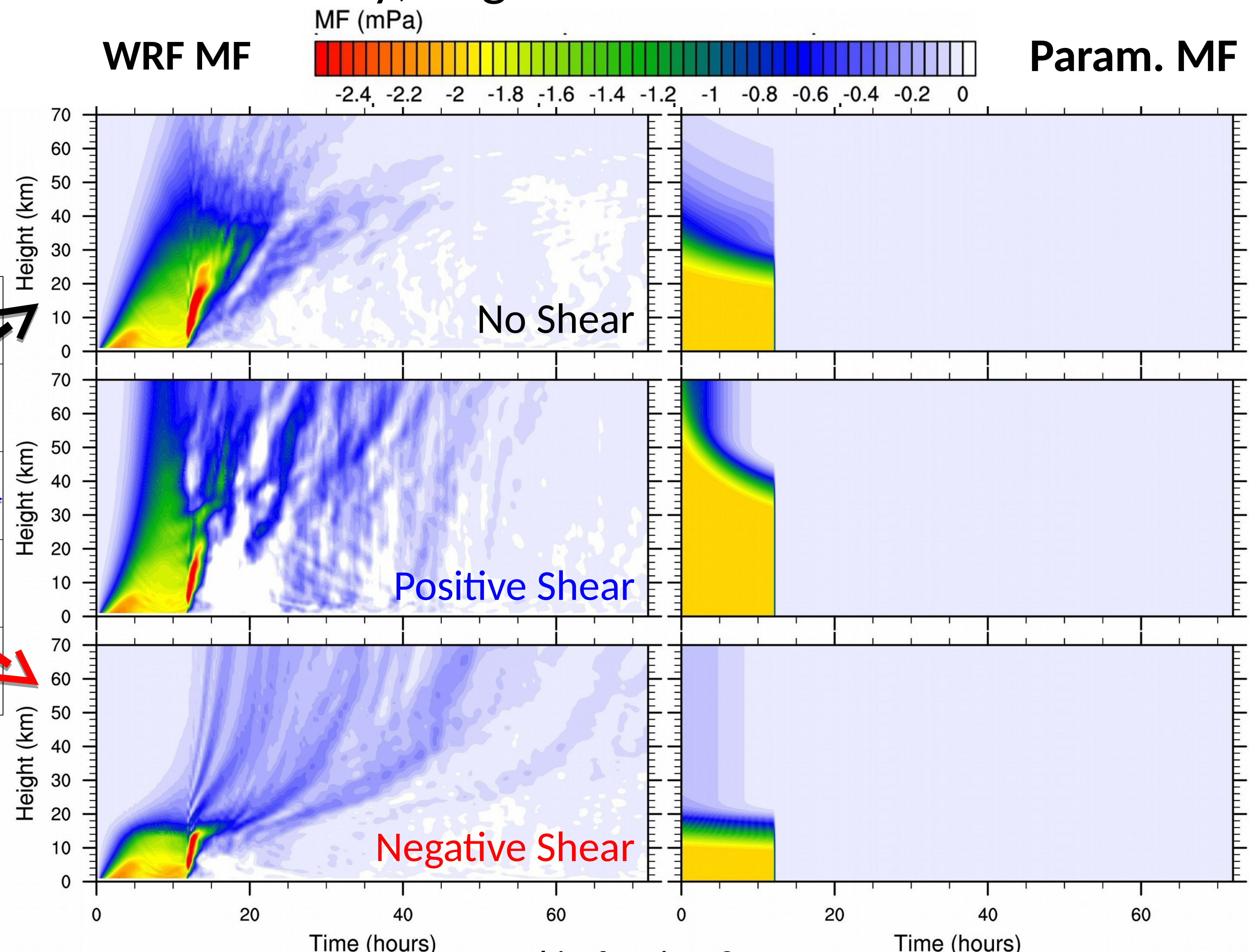
  - Applied to parameterized wave over inner 200 km “grid cell” for amplitude
- Parameterized momentum deposition applied to entire domain width
- That is, same MF out of domain as WRF and same initial momentum profile as WRF



# WRF, Parameterization MF Comparison

- Parameterization: No delay; larger MF aloft



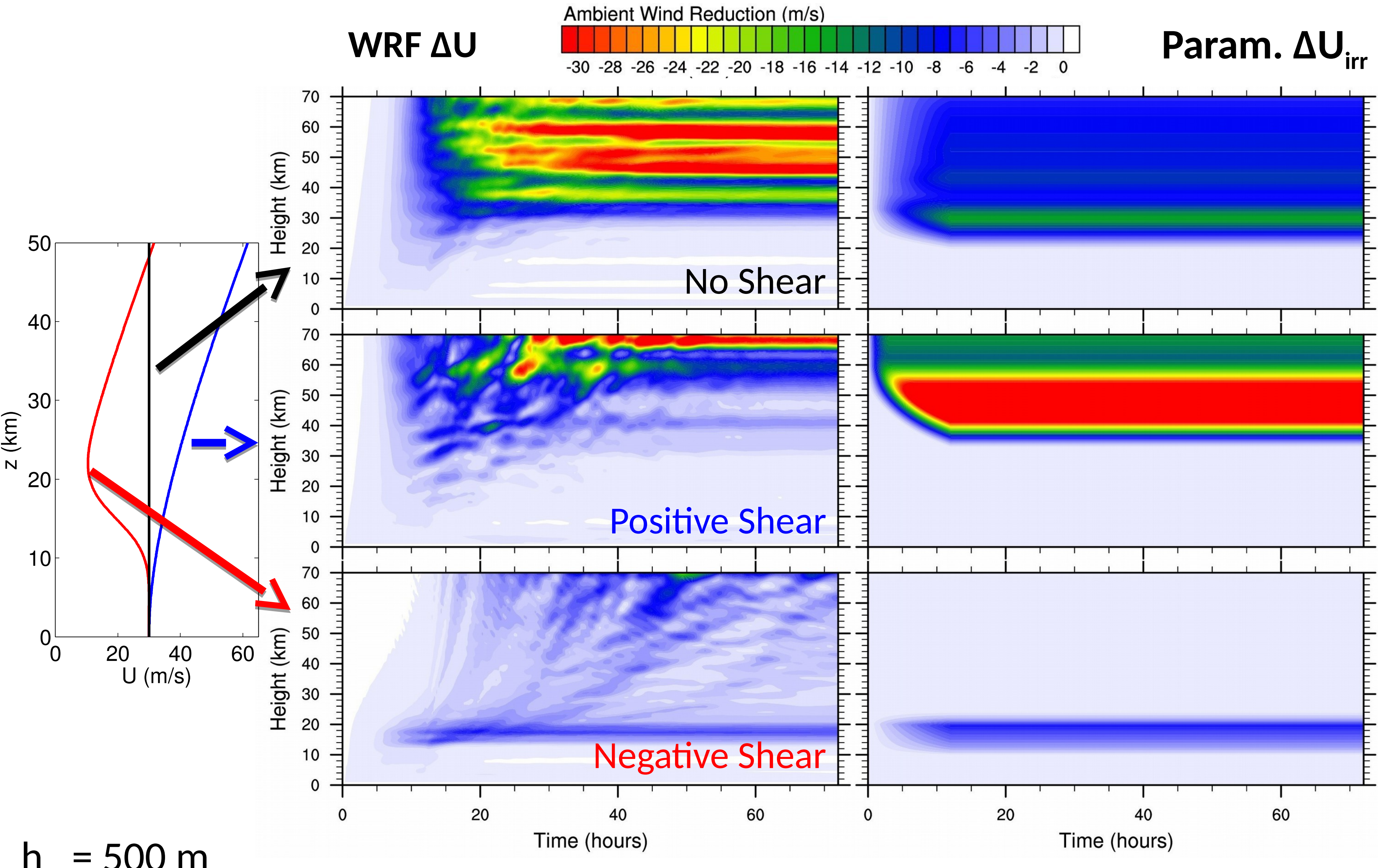




$h_m$ = 500 m

Skip for time?

# WRF, Parameterization $\Delta U_{irr}$ Comparison

- Substantial $\Delta U_{rev}$ (5-10 m/s) prior to breaking




$h_m$ = 500 m

# Sat. MF Deposition: Dependent on Shear

- Saturation Assumption results in MF deposition dependent upon ambient vertical wind shear:

$$MF_s = -\frac{\overline{\rho} k}{2N} U^3$$

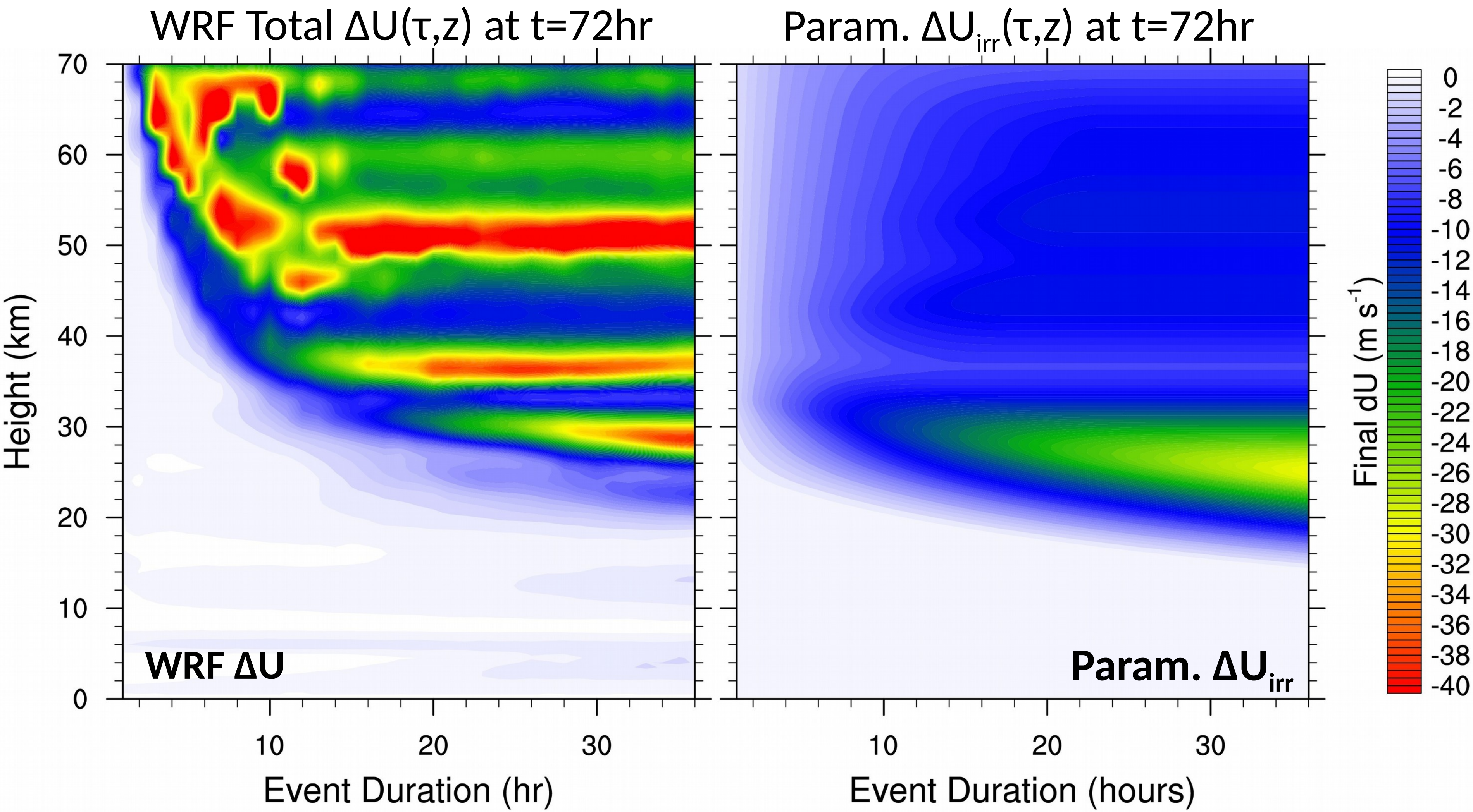
$$\frac{dMF_s}{dz} = -\frac{k}{2N}\left(\underbrace{3\overline{\rho}U^2\frac{dU}{dz}} + U^3\frac{d\overline{\rho}}{dz}\right)$$

- Negative shear develops at $z_{break}$, which causes stronger momentum deposition, further increasing shear, …
  - Solution blows up after some time; **strongly dependant upon vertical resolution**
- Apply 10-km ($\approx\lambda_z$) vertical running average smoother to MF force a dissipation scale
- This allows downward communication of attenuation, descending critical level dynamics

Probably skip for time

# Influence of Event Duration

- Lowest dissipation level decreases with increasing event duration
  - Longer durations allow fuller wave spectrum aloft, increasing wave amplitudes
- Monochromatic, instantaneous parameterization assumptions eliminate this




No Shear, $h_m$ = 500 m

# Conclusions

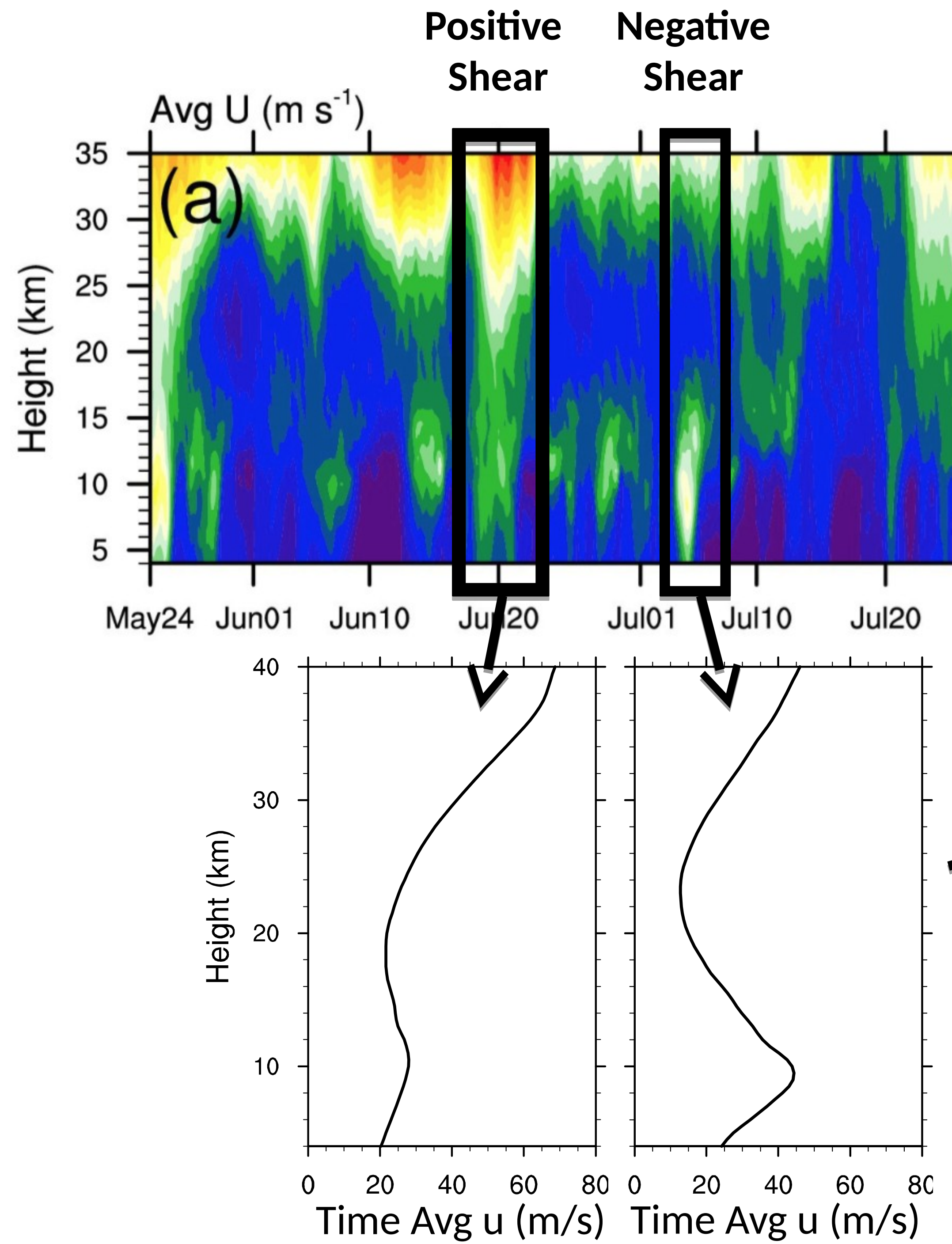
- A finite duration MW forcing causes non-dissipative vertical gradients in MF $\Delta U_{rev}$
- $C_{gz}$ spectrum controls spectral evolution aloft (at least initially)
  - Spreads MF profiles vertically, impacts $\Delta U_{rev}$
  - Causes temporally asymmetric response in linear cases; can take days to recover because of slow long waves
- $\Delta U_{rev}$ can be substantial (5-10 m/s) prior to wave breaking; $\Delta U_{irr}$ dominates, increases with event duration/impulse
- Parameterization errors are large, dependent upon ambient wind profile

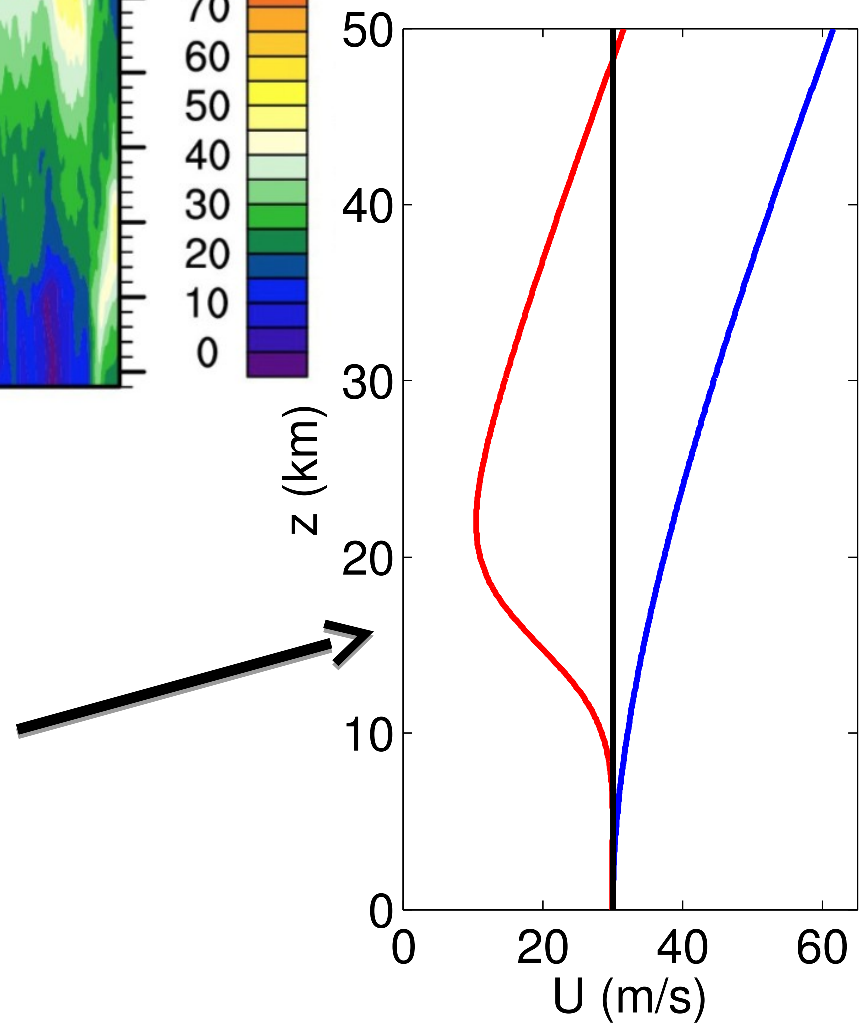
# Parameterization Comments

Parameterization Assumptions

1. Instantaneous
   – Could be relaxed, but only useful if 2. relaxed as well
2. Monochromatic
   – Could be relaxed, but only useful if 1. relaxed as well
3. Steady Background ($\Delta U_{rev}$=0)
   – Might give more accurate breaking levels if relaxed
4. Saturation Assumption
   – Can under or over predict MF deposition significantly depending on ambient wind profile!

- Think relaxing 1. and 2. together will result in better performance. Applying saturation spectrally is tricky.



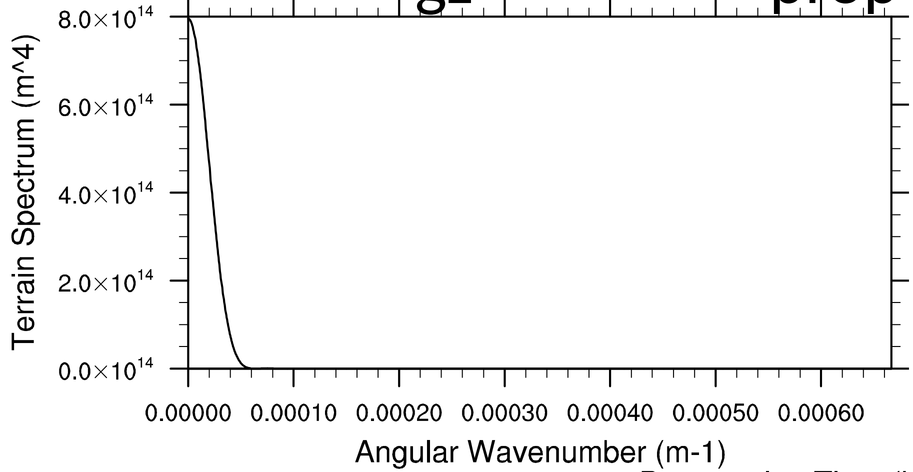




**Idealized Profiles** guided by actual stratospheric profiles

# WRF, Param Domain Momentum Reduction

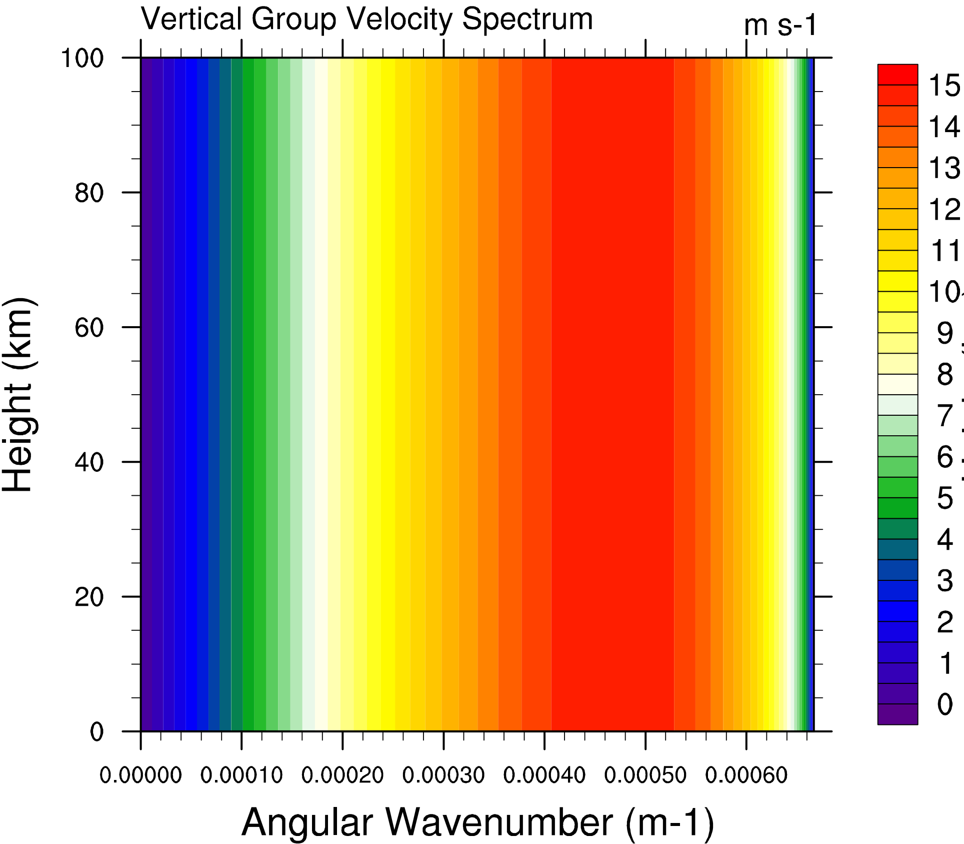
- Want to make time series plots of domain (x and z) integrated x-momentum for comparison.

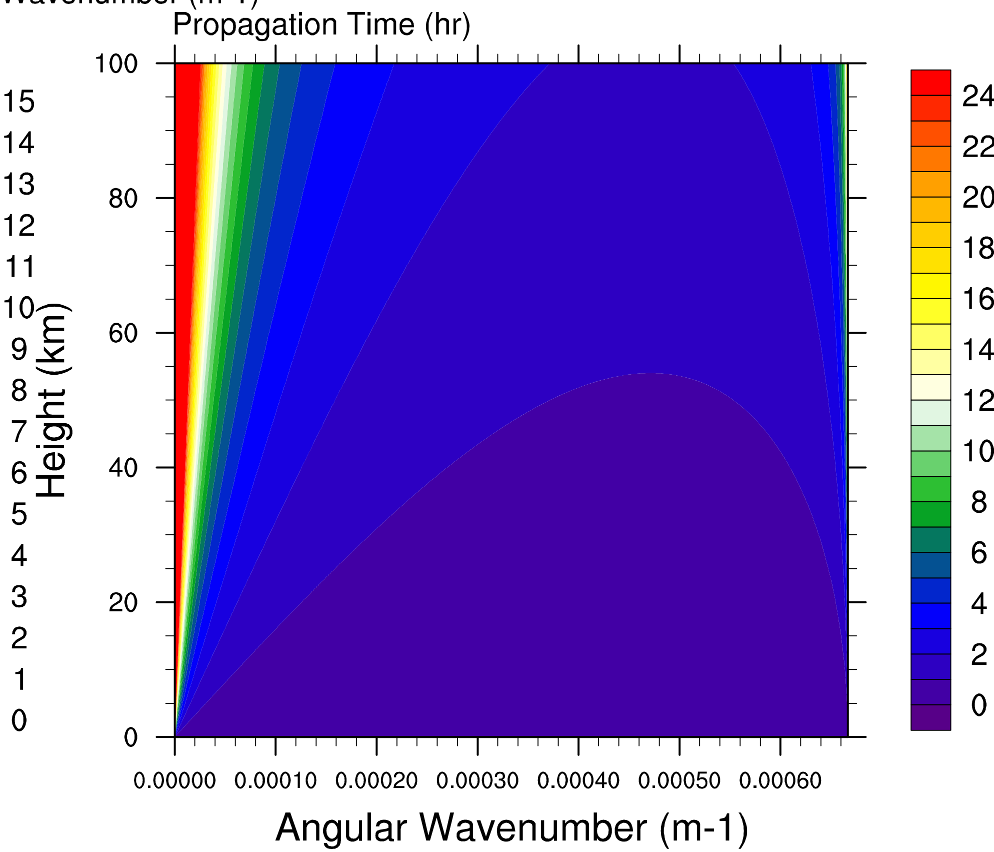
# No Shear $c_{gz}(z,k)$, $t_{prop}(z,k)$



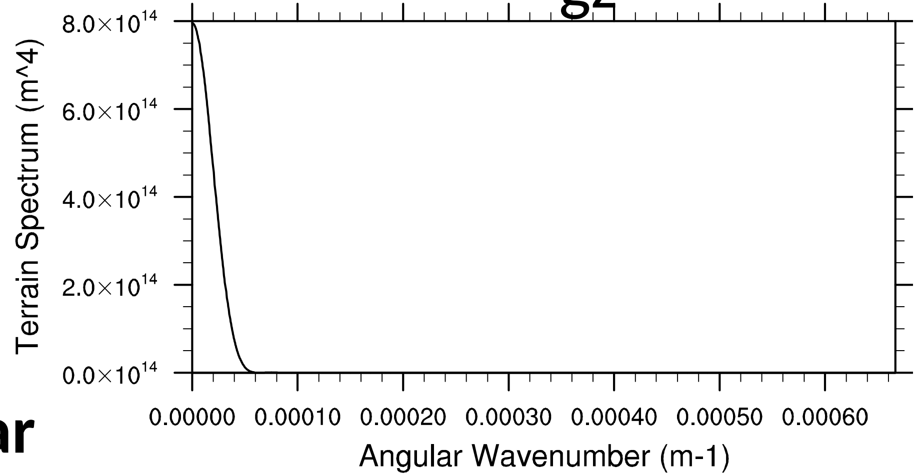

k up to buoyancy cutoff

## No Shear



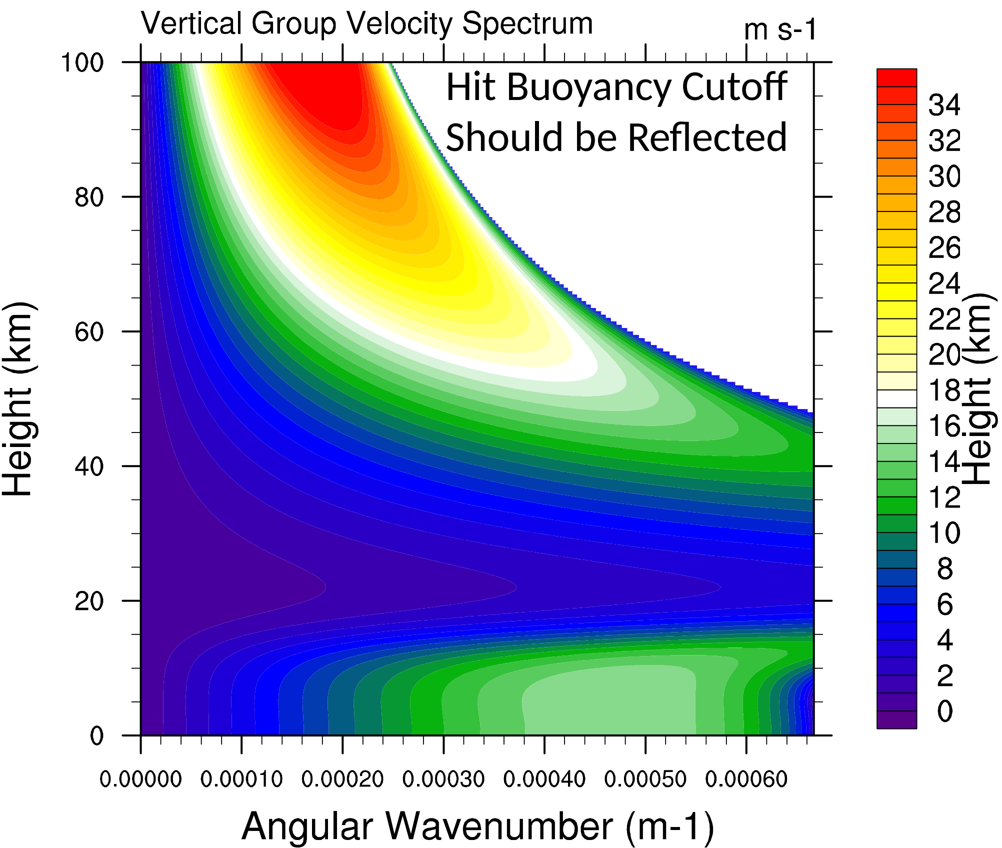



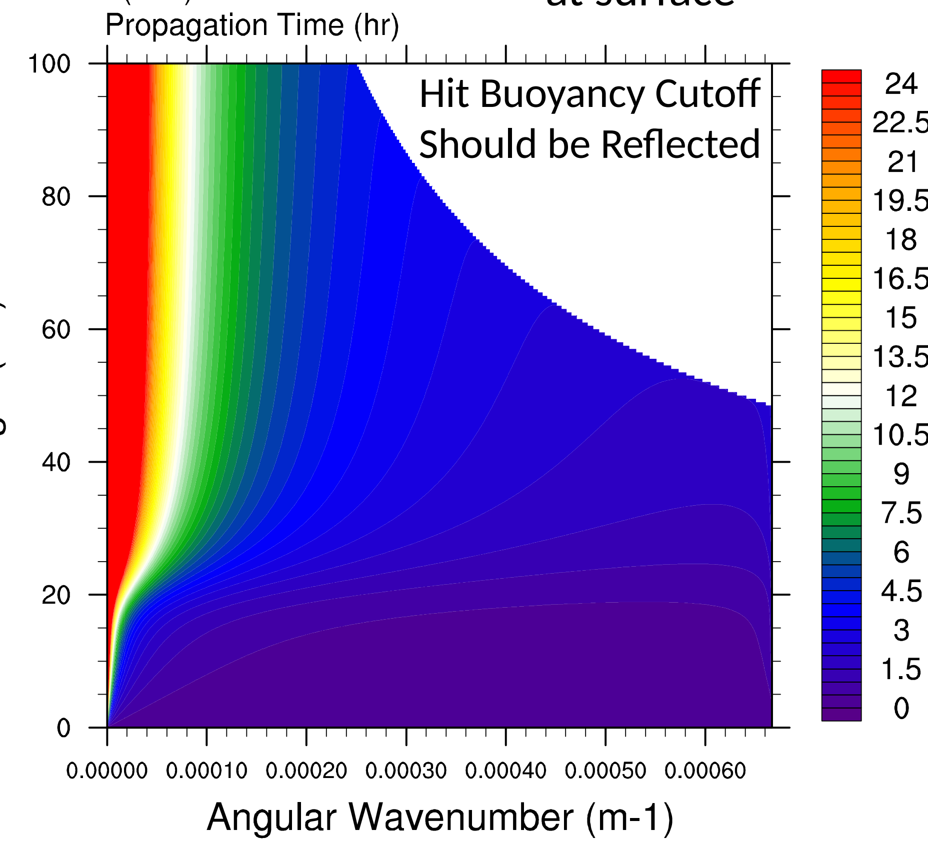
# Negative Shear $c_{gz}(z,k)$, $t_{prop}(z,k)$



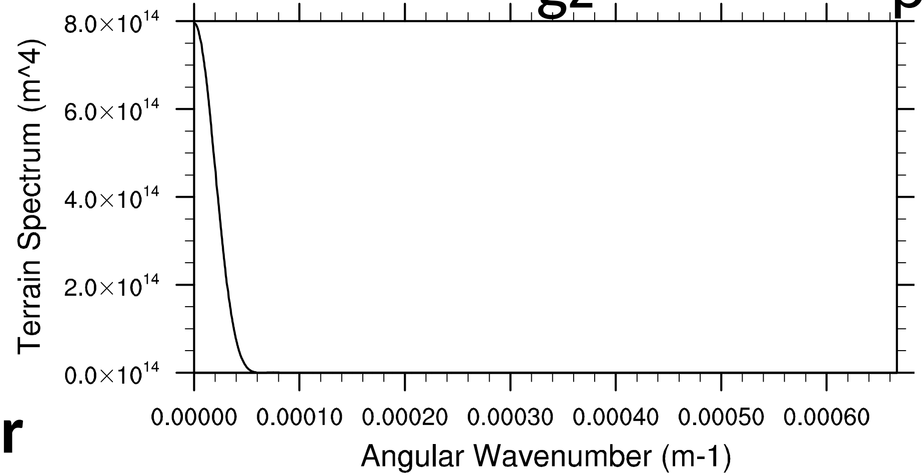

k up to buoyancy cutoff at surface

**Negative Shear**



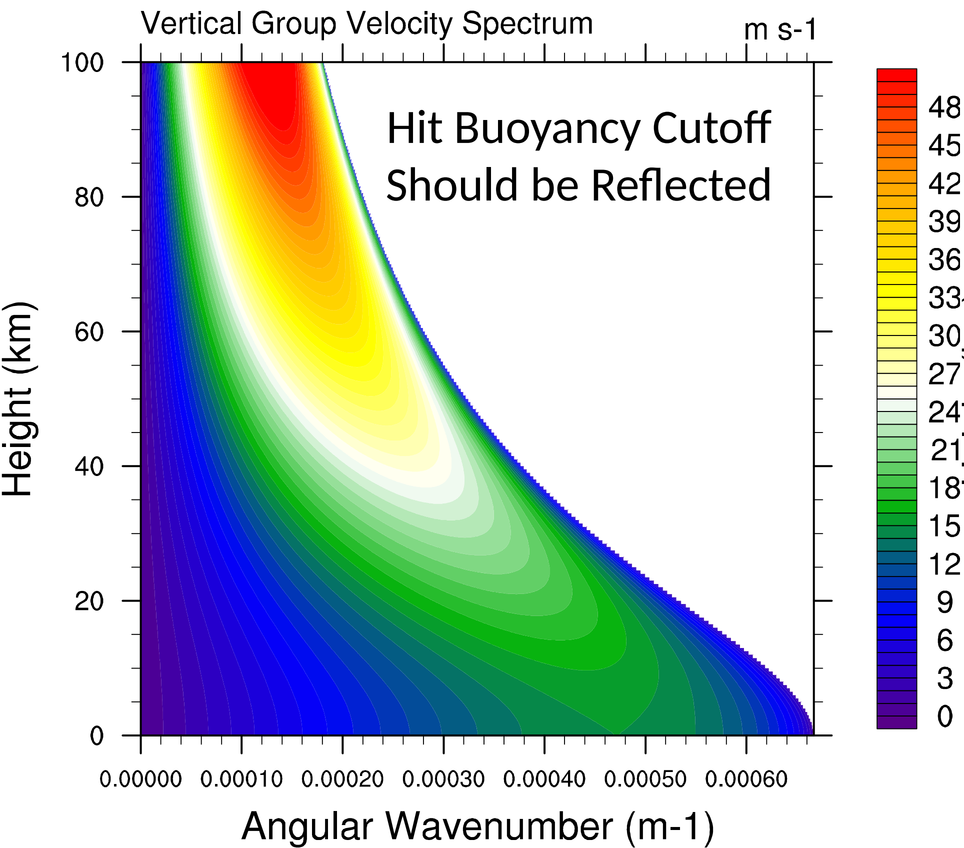



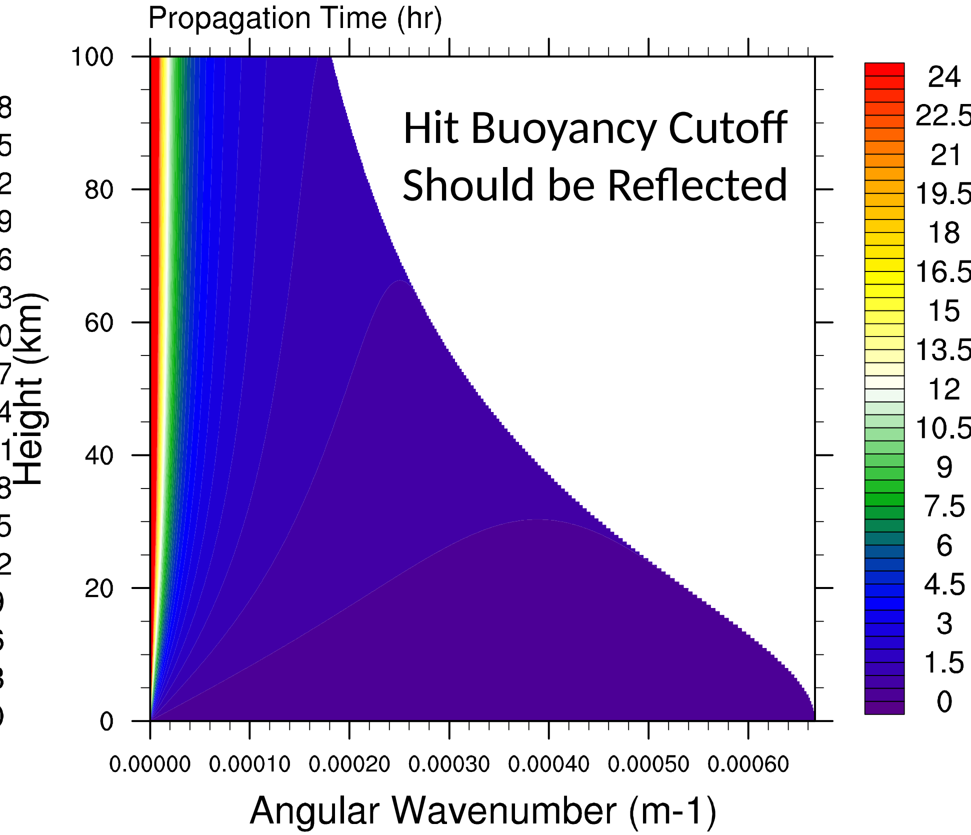
# Positive Shear $c_{gz}(z,k)$, $t_{prop}(z,k)$

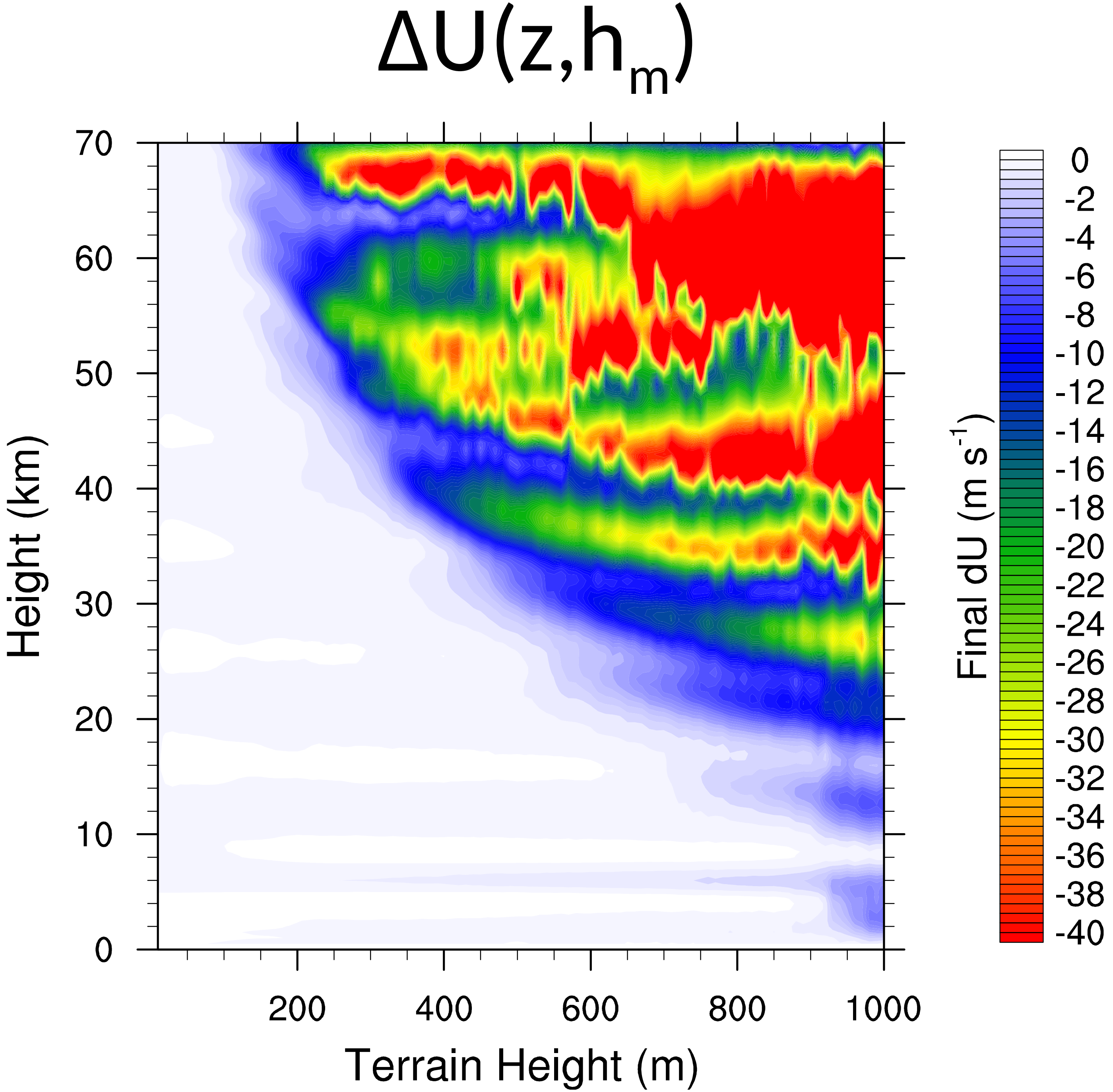



## Positive Shear

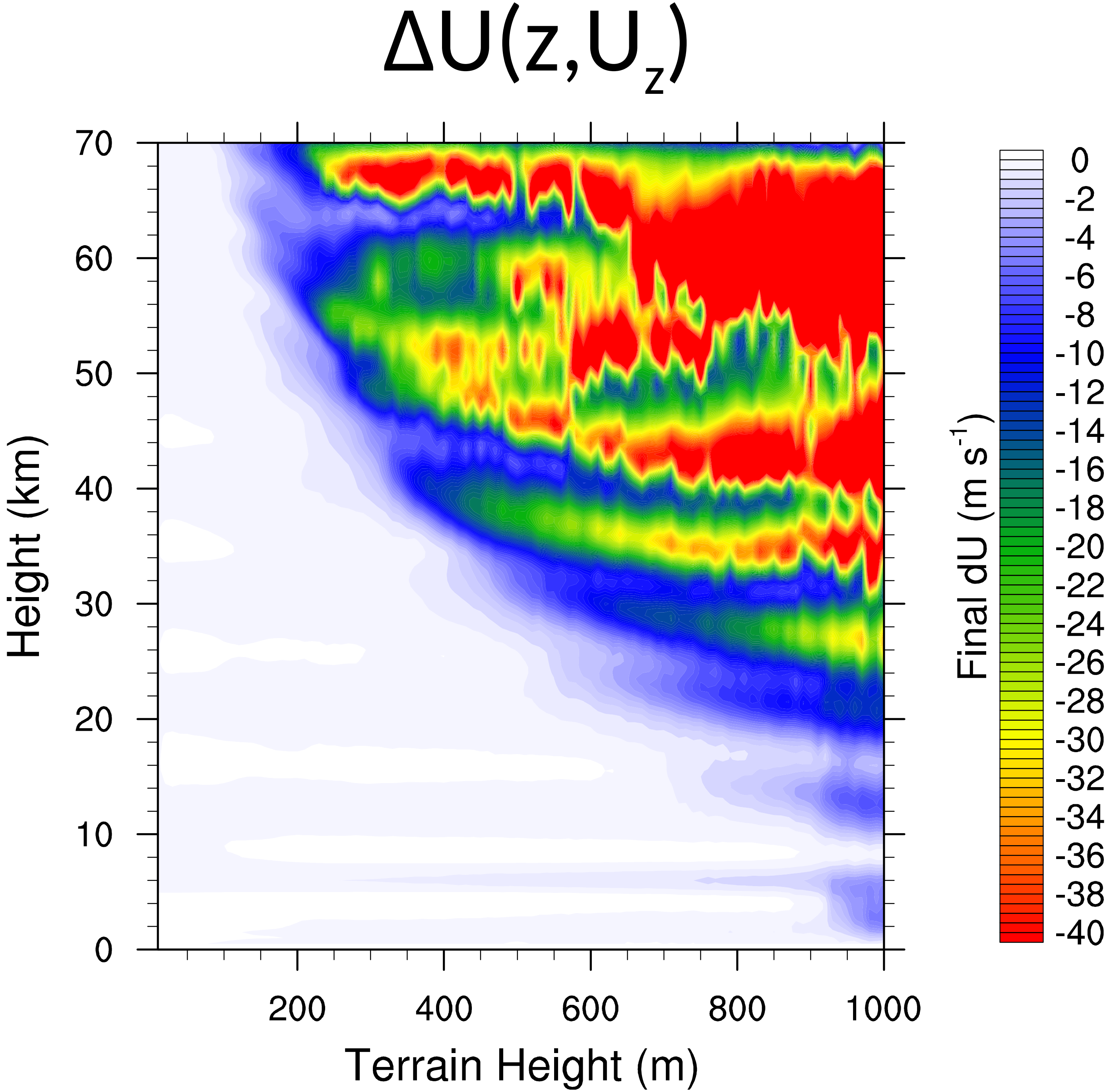
k up to buoyancy cutoff at surface





ΔU(z,h_m)
Height (km)
Terrain Height (m)
Final dU (m s-1)
0
10
20
30
40
50
60
70
200
400
600
800
1000
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24
-26
-28
-30
-32
-34
-36
-38
-40


ΔU(z,U_z)
Height (km)
70
60
50
40
30
20
10
0
200
400
600
800
1000
Terrain Height (m)
Final dU (m s-1)
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24
-26
-28
-30
-32
-34
-36
-38
-40

# ΔU(z,Valve Min)