Multi-Scale Dynamics of Gravity Waves

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and many others:

https://ms-gwaves.iau.uni-frankfurt.de/index.php
State of the Art: GW Impacts

Gravity-wave effects numerous, e.g.
- Clear-air turbulence (e.g. Koch et al 2005)
- Clouds (e.g. Zhang et al 2001, 2003, Joos et al 2009)
- Middle-atmosphere waves (QBO, solar tides, PWs)
- residual circulation
  - GW impact in stratosphere (e.g. Palmer et al 1986)
  - GW control in mesosphere (e.g. Lindzen 1981)
- Indirectly: Impact middle atmosphere on troposphere (downward control)

Scaife et al (2005)
State of the Art: Parameterization of GW Processes

Sources:
- **Orographic GWs best understood** (Palmer et al 1986, Jiang et al 2002)
- **Convective GWs** (Chun & Baik 1998, Beres et al 2005, Song & Chun 2005, …)
- **Spontaneous GW emission** (e.g. Plougonven & Zhang 2014)
- **Secondary waves, …**
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GW propagation

- **Simplifications** of WKB Theory (Grimshaw 1975, Achatz et al 2017) for efficiency: **Single-column** and **steady state** limit validity (e.g. Bühler & McIntyre 2003, Ribstein & Achatz 2016, Bölöni et al 2016)
- **Synoptic-scale balanced background assumed**
  But NWP models resolve some GWs!
- **GW propagation through sharp gradients**: **Tropopause**
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  *Single-column* and *steady state* limit validity
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- **Synoptic-scale balanced background assumed**
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- **GW propagation through sharp gradients: Tropopause**

GW dissipation:
- **Saturation** (Lindzen 1981, …)
  not in agreement with DNS
- **Wave-mean flow interaction** (Dosser & Sutherland 2011, Bölöni et al 2016)
Objectives: Central Goals & Key Research Areas

Goals
1. Efficient parameterization based on understanding and computational representation of GW processes
2. A prognostic model for SGS GWs & implementation into NWP and climate model.

Key Research Areas

GW spatial, temporal and spectral Distribution

GW Processes: Sources, propagation, dissipation

Theory ↔ Modelling ↔ Measurements ↔ Experiments

GW Impacts
Modelling Measurements Experiments
D1: Analysis of measurements and weather-service data of the GW distribution

Some examples: Overlap DEEPWAVE & field campaign

- Refraction of GWs into the polar night jet (Ehard et al 2017)
- Mountain waves New Zealand (Portele et al 2017, subm.)
- Field campaign northern Scandinavia winter 2015/16
D1: Analysis of measurements and weather-service data of the GW distribution

Some examples: RMR lidar Kühlungsborn

- Climatology T variances (K. Baumgarten et al 2017)

- Unprecedented long data set (4-13 May 2016) (K. Baumgarten et al 2017, subm.)
D1: Analysis of measurements and weather-service data of the GW distribution

Some examples: Satellite data

- Global GW momentum fluxes from AIRS data (Ern et al 2016)
Progress & Results: GW Distribution

D2: Non-hydrostatic GW permitting/resolving global model (UA-ICON with MS-GWaM)

Upper-Atmosphere-ICON with standard GW parameterizations (Borchert et al 2017, in prep.)

- height dependence of $g$
- Coriolis acceleration for all spatial directions
- sphericity changes grid volumes with height
- Development completed (test case and NWP-scores show good results)
Progress & Results: GW Distribution

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Multi-Scale-Gravity-Wave Model (WKB model)

- implemented (1D, interactive)
- validation in planning (Bölöni et al 2017, in prep.)
Progress & Results: GW Distribution

D3: Validation of the GWs simulated by UA-ICON

Activities so far:

- Implementation of **observational filter** for comparisons against satellite data
- ICON simulations of **campaign episodes** (DEEPWAVE, northern Scandinavia Jan 2016)
Progress & Results: GW Processes

P1: GW source processes and their efficient parameterization

Results so far:

- Tuning of convective GW-source parameterization (Thrinh et al 2016)
Progress & Results: GW Processes

P1: GW source processes and their efficient parameterization

Results so far:

- Demonstration/analysis of spontaneous imb. in differentially heated rotating annulus *(Hien et al 2017, revised)*
- ... uses wave analysis tool UWADI *(Schoon & Zülicke 2017, subm.)*

![Diagram showing GW processes](image)
Progress & Results: GW Processes

P1: GW source processes and their efficient parameterization

Results so far:
• GWs in the differentially heated annulus (Rodda et al 2017, subm.)
Progress & Results: GW Processes

P2: GW-mean-flow interactions & Multi-Scale Gravity-WAve Model (MS-GWaM)

Some Results:
- Generalized theory: all stratifications, nonlinear, GMs (Achatz et al 2017)
- Comparsion role direct GW-mean-flow interaction with turbulence (Bölöni et al 2016)
- Impact lateral propagation on tides (Ribstein et al 2015, Ribstein & Achatz 2016)
- Interaction sub-mesoscale waves with mesoscale flow (Wilhelm et al 2017, in prep.)
- GW-tropopause interactions (Gisinger et al 2017, subm., Pütz et al 2017, subm.)
Progress & Results: GW Processes

P3: GW Dissipation

Results:
- Travelling-wave solutions to modulational equations *(Schlutow et al 2017)*
- Stability analysis
Focus:

Wave-mean-flow Interaction beyond traditional parameterization approaches
Classic WKB (Grimshaw 1975, ... for illustration 1D:

Locally monochromatic fields of the form:

\[ b'(x, t) = \Re \ B(z, t) e^{i\phi(x,t)} \]

Local wavenumber and frequency:

\[ k(z, t) = ke_x + me_z = \nabla \phi, \quad \omega(z, t) = -\partial \phi/\partial t \]

Wave-action density so that (e.g.)

Wave-action density \( A(z, t) \) so that (e.g.)

\[ E_{GW}(z, t) = A(z, t) \hat{\omega}(m) \]

Along rays, defined by

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\[ \frac{dz}{dt} = c_g \]

\[ \frac{dm}{dt} = -k \frac{\partial U}{\partial z}, \quad \frac{dA}{dt} = -A \frac{\partial c_g}{\partial z} \]

Mean flow: \[ \frac{\partial U}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} u'w') = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (c_g k A) \]
Ray tracing with caustics: Stability Problem

GW packet refracted by a jet

Rieper et al (2013)
Ray tracing with caustics: Uniqueness Problem

**Locally monochromatic fields**

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Crossing rays (caustics): uniqueness problem for \( A \) and \( m \)!
Ray tracing with caustics: examples for caustic situations

Nonuniqueness of wave number and wave-action density arises easily:

e.g. by wave-induced mean flow
Ray tracing with caustics: spectral approach

Linear limit: wave field can be decomposed into fields with single-valued wavenumbers.


Phase space wave-action density

$$ N(m, z, t) = \int d\alpha A_{\alpha}(z, t) \delta[m - m_{\alpha}(z, t)] \quad \Leftrightarrow \quad A(z, t) = \int dm N(m, z, t) $$

This satisfies the conservation equation

$$ \frac{\partial N}{\partial t} + \frac{\partial}{\partial z} \left( c_g N \right) + \frac{\partial}{\partial m} \left( m N \right) = 0 \quad m = -k \frac{\partial U}{\partial z} $$

Mean flow:

$$ \frac{\partial U}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \left( \bar{\rho} u' w' \right) = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \left( \int dm c_g k N \right) $$

Generalization to 3D straightforward.
Ray tracing with caustics: efficient numerics (Muraschko et al 2015)

Phase-space velocity is non-divergent

\[
\frac{\partial c_g}{\partial z} + \frac{\partial m}{\partial m} = \frac{\partial}{\partial z} \frac{\partial \Omega}{\partial m} + \frac{\partial}{\partial m} \left( - \frac{\partial \Omega}{\partial z} \right) = 0
\]

hence
- flow is volume preserving
- rays cannot cross
- wave-action density conserved on rays
- wave-action density conserved on rays
- region of nonzero approximated by rectangular ray volumes
- ray volumes move with central ray
- ray volumes change height (\(\Delta z\)) and width (\(\Delta m\)) in area-preserving manner
Ray tracing with caustics: efficient numerics (Muraschko et al 2015)

**hydrostatic wave packet**  
(Boussinesq)

Rays are no wavepackets

No turbulence taken into account!
Ray tracing with caustics: no numerical instabilities (Bölöni et al 2016)

GW packet refracted by a jet

LES

WKB ray tracer

WKB finite volume
Direct wave-mean-flow interaction: comparison with role of wave breaking

- transient GWs can interact with the mean flow without the onset of turbulence (eg Dosser & Sutherland 2011)
- GW parameterizations (steady-state approximation) only rely on wave breaking

Comparative role of wave transience (direct interaction) vs wave breaking?
direct wave-mean-flow interaction vs wave breaking (Bölöni et al 2016)

horizontally infinite GW packets in interaction with mean flow

- 1D: $U(z, t), A(z, t), m(z, t)$
- direct GW-mean-flow interaction always active
- WKB:
  - WKB: $E_{\text{mean}} + E_{\text{wave}} = \text{const.}$

tools:
- wave resolving LES (reference data)
- fully coupled WKB (reference data)
- turbulence onset
  - once static instability threshold can be surpassed
  - parameter accounting for phase cancellations between spectral components
  - (scale selective) eddy viscosity/diffusivity reduces wave amplitude to inst. threshold
  - parameter $\alpha \in [1, 2]$ accounting for phase cancellations between spectral components
  - (scale selective) eddy viscosity/diffusivity reduces wave amplitude to inst. threshold
direct wave-mean-flow interaction vs wave breaking (Bölöni et al 2016)

static instability hydrostatic wave packet

**LES** (wave-resolving)

**WKB**

- Integrated energy
- Altitude (km)
- Time (N x t)

green: wave energy
blue: mean flow energy
red: sum
direct wave-mean-flow interaction vs wave breaking (Bölöni et al 2016)

static instability hydrostatic wave packet
direct wave-mean-flow interaction vs wave breaking (Bölöni et al 2016)

static instability non-hydrostatic wave packet

LES (wave-resolving)

WKB with saturation (turbulence param.)
direct wave-mean-flow interaction vs wave breaking (Bölöni et al 2016)

static instability non-hydrostatic wave packet

LES (wave-resolving)

WKB with saturation (turbulence param.)

steady-state (GW parameterization)
Large-scale waves forced by the diurnal cycle of solar heating
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Two components:
- **Migrating tides**
  follow solar movement
Solar tides

Large-scale waves forced by the diurnal cycle of solar heating

Two components:
- **Migrating tides** follow solar movement
- **Nonmigrating tides**: all the rest
Solar tides

Large-scale waves **forced by the diurnal cycle of solar heating**

**Two components:**
- **Migrating tides**
  follow solar movement
- **Nonmigrating tides:**
  all the rest

**Interaction with GWs:**
- STs influence **GW propagation and amplitude development**
- GW impact on STs by **GW momentum and buoyancy deposition**
Tidal model in interaction with GWs (Ribstein et al 2015, Ribstein & Achatz 2016)

From GCM data (HAMMONIA, Schmidt et al 2006):
- Seasonally dependent reference climatology
- Diurnal heating cycle

\[ \mathbf{u}(\lambda, \phi, z), \; \mathbf{T}(\lambda, \phi, z) \]
\[ R \sum_n Q_n(\lambda, \phi, z) e^{i n \omega t} \]

Linear model (Achatz et al 2008, based on KMCM, Becker and Schmitz 2003)

\[ \mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'(\lambda, \phi, z, t) \]
\[ T = \bar{T} + T'(\lambda, \phi, z, t) \]

GW fluxes from 4D WKB model with rays propagating on

First implementation of a fully coupled transient ray tracer into a global model

GW fluxes from 4D WKB model with rays propagating on \((\bar{\mathbf{u}} + \mathbf{u}', \bar{T} + T')\)

First implementation of a fully coupled transient ray tracer into a global model
Tidal model in interaction with GWs
(Ribstein et al 2015, Ribstein & Achatz 2016)

\[ \frac{dx_n}{dt} = \rho \nabla_n (\tilde{u} + u') \]

\[ \frac{dz}{dt} = \nabla_z (\tilde{v} + v') \]

\[ \frac{dk_b}{dt} = \nabla_b (\tilde{u} + u') \cdot \nabla_b (\tilde{v} + v') \]

\[ \frac{dm}{dt} = -k \frac{d}{dz} (\tilde{u} + u') - l \frac{d}{dz} (\tilde{v} + v') \]

3D effects (beyond single column)

- Horizontal GW propagation
- Horizontal gradients in reference climatology and tides
- Horizontal gradients in reference climatology and tides
- Horizontal GW flux convergence

\[ \left( \frac{\partial}{\partial t} + \bar{u} \cdot \nabla_h \right) T' + \nu' \cdot \nabla T + \cdots = 0 \sum_n Q_n(\lambda, \phi, \rho) e^{in\omega t} \]

\[ - \nabla_h \cdot \left( \rho \frac{\partial}{\partial t} \left( \rho \mathbf{u}_{GW} \mathbf{T}_{GW} \right) \right) \]

\[ - \nabla_h \cdot \left( \rho \mathbf{u}_{GW} \mathbf{u}_{GW} \right) \]
Tidal model in interaction with GWs (Ribstein et al 2015, Ribstein & Achatz 2016)

3D effects (beyond single column)

zonal-mean daily-mean GW forcing (December)
Tidal model in interaction with GWs (Ribstein et al 2015, Ribstein & Achatz 2016)

3D effects (beyond single column)
3D effects (beyond single column):

- **Tidal model**
  - Diurnal forcing only induces diurnal tide
  - GWs can induce semi-diurnal tide

\[
\frac{d\mathbf{y}'}{dt} = \mathbf{L}(\mathbf{u}', \mathbf{T})\mathbf{y}' + \Re \sum_n Q_n e^{-in\Omega t} + \mathbf{F}_{GW}(t)
\]

\[||V||_{\text{non-migrating semi-diurnal}}^{\text{semi-diurnal}} \text{ [m/s] in December}\]
Tidal model in interaction with GWs (Ribstein et al 2015, Ribstein & Achatz 2016)

3D effects (beyond single column):

- **Tidal model**
  - Diurnal forcing only induces diurnal tide
  - GWs can induce semi-diurnal tide (40% effect)
  - GWs can induce semi-diurnal tide (40% effect)
Summary

- **Approximations in present-day GW parameterizations critically limit their validity**
  - Single-column
  - Steady state
- **First implementation of a generalized approach into a global model**
- **Significant impact:**
  - Zonal-mean forcing
  - Solar tides


G WAVES
https://ms-gwaves.iau.uni-frankfurt.de/index.php

- Investigation multi-scale dynamics of GWs in 6 projects
- prognostic WKB GW parameterization to be developed for NWP and climate model
- To be addressed:
  - Sources
  - Propagation
  - dissipation
- Combined effort:
  - Theory,
  - modelling,
  - measurements,
  - laboratory experiments